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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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AEROPLANE TOPOGRAPHIC SURVEYS

By George T. Bergen,* M. Am. Soc. C. E.

Synopsis

The object of this paper is to acquaint engineers with a method of making accurate contour maps directly from aerial photographs. The process discussed is the product of research and development since the World War. Some important elements of the problem are stated, followed by a brief exposition of the means adopted for a solution.

The control needed is confined to a few ground elevations for each photograph and a small number of distance measurements for each survey. The resulting maps are available in a fraction of the time necessary for surveys by old methods. Their accuracy often exceeds that of comparable ground surveys.

The method is applicable to a wide range of conditions. It offers marked advantages in making maps of large areas for engineering studies and preliminary locations, particularly in country of bold relief and considerable growth of timber or brush.

Introduction

Aerial photography is a relatively new resource of the surveyor and map maker. Extravagant claims have doubtless been made for it, as is not infrequent with new processes. On the other hand, its scope has been widened and its use adopted sufficiently to indicate that it has come to stay. Its limits are by no means reached. "In future mapping the use of aircraft will play an important part."†

The speed of the aeroplane, the rapidity and detailed record of the camera, suggest to any engineer tremendous advantages for a process by which accurate topographic maps can be produced from aerial photographs, with relatively little instrumental ground measurement required for control and correction of the pictures. The photographs themselves cannot ordinarily be used as maps, because of the variation in scale. Particularly in country having any substantial relief, the horizontal scale of pictures varies to an extent not per-

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Note.—Written discussion on this paper will be closed in August, 1926. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

* Civ. Engr., Day & Zimmermann, Inc., Philadelphia, Pa.

† Report of September 30, 1919, to the President by Conference of Representatives of Federal Map-Making Organizations, including W. M. Black, Maj-Gen., U. S. A. (*Retired*), M. Am. Soc. C. E. (then Chief of Engineers, U. S. A.), William Bowie, M. Am. Soc. C. E., Chief, Division of Geodesy, U. S. Coast and Geodetic Survey, George Otis Smith, Director, U. S. Geological Survey, A. P. Davis, Past-President, Am. Soc. C. E. (then Director, U. S. Reclamation Service), Thomas H. MacDonald, Chief of Bureau of Public Roads, O. C. Merrill, M. Am. Soc. C. E., Chief Engineer, Forest Service, and eight others.

missible for engineering purposes. In any event, a single photograph furnishes no elevations.

Although aerial mapping is new, the use of photography in surveying is not. Most engineers have some acquaintance with the theory, if not the practice, of the photo-theodolite. Records are available of the scientific use of photography in surveying as early as 1850. European army officers at that time made use of the camera somewhat after the present manner of using the transit. At the end of measured base lines, pictures were taken on vertical planes, and the angles between base lines and optical axes determined, thus locating objects by triangulation. Photographs were later made from balloons, some by cameras having several lenses and picture planes simultaneously exposed, so as to record more information and supply some check on the position of the viewpoint.

Developments of the World War revolutionized the use of aerial photography. The changes were so radical in their nature and the uses so extensive as to give the concept of something wholly new. André Carlier, Président de l'Association Française Aérienne, is authority for the statement that, during the last years of the war, 80% of the information about the enemy was due to aerial photography.*

An important war development was the mosaic of aeroplane photographs. A series of views were pieced together to form a battle map, constantly subject to replacement by a flow of new photographs showing everchanging details.

After the war, attention was soon directed toward the development of peaceful uses for this new resource. Numerous manufacturers engaged in the commercial production of photographic mosaics. Because of the rapidity with which they can be obtained and the vivid conception they give of a large area, they have many uses. A mosaic, however, is subject to severe scale variation, particularly where it covers ground of marked relief.

Various efforts have been made to bridge the gap between photographs and maps. One method is to use the picture for what it is worth as a step in mapping by old methods. Individual pictures, mosaics, or some derivatives of these have thus been used simply as plane-table sheets. Having before him the wealth of detail shown by the photographs, the topographer is aided and an economy is effected in ground surveys, particularly in making small-scale maps of flat country for general purposes. In such a case, the absence of relief and the reduction of scale attending the processes through which the map assumes final form, minimize the scale discrepancy and render possible considerable use of the photographic information. In such methods, all measurements of elevation and the work of locating contours are done on the ground in the old manner; aerial photography is merely an auxiliary.

The ideal method will reverse this relation, will make maps directly from the camera records, and dispense with ground measurements or reduce these to a minimum needed for control only. One effort toward this goal uses the three-point method of control; that is, if the distances between three points of equal elevation are known and their images clearly distinguishable on the

* "Tout le monde à présent a l'esprit des services qu'elle rendit pendant la guerre, et il n'est pas exagéré de dire que, dans les dernières années, 80% des renseignements sur l'ennemi étaient donnés par elle," "La Photographie Aérienne", *L'Illustration,* 13 décembre, 1924.

photograph, it is possible by computation to approximate the picture plane. This method has serious objections. Many such triangles must be located on the ground, involving much field work. The computation is laborious, the result unsatisfactory. In practice it is possible to make an indefinite number of re-projected positions answer to the calculated size and position of triangle. Often it is difficult or impossible to locate a suitable triangle with points of equal elevation. Resort may be had to points with measured differences of elevation, but the computation is then made more laborious. In any event, there remains the problem of locating contours.

Such considerations have led to efforts to evolve a practical method for producing directly from the data in the photographs an accurate contour map, without excessive work on the ground. Although the writer is not directly connected with the development of the particular process to be described, he has had unusual opportunity to become familiar with it, having had occasion to check certain work done by it and to study closely the operations and instruments it involves.

The Problem

It is not to be expected that an area of marked relief will ever be reproduced directly with the attributes of a good map. With proper equipment photographs can be made into records of high precision, but they follow certain laws inapplicable to maps. To convert the data recorded in pictures to the form of maps requires analysis and changes to meet these inherent differences as well as to overcome the effects of practical working obstacles. Distances are represented on maps unaffected by the point of view; on photographs, even a series of aerial views from a great altitude, points and lines follow the phenomenon of parallax, suffering changes in their apparent positions with changes of the point of view. A variation in the height of the aeroplane and camera changes the photographic scales. Efforts to hold aerial cameras truly vertical have failed, and the tilt causes further displacements of points in the photographs, varying between succeeding views and within a single picture.

In short, to convert the data of photographs to a uniform scale a variety of obstacles and limitations must be met, some by nature inherent in all photographs, some characteristic of photographic materials, some due to difficulties met in practical operation, but the most troublesome are those involving mechanical obstacles.

Limitations Due to Nature of Photographs

Pictures are conic projections.* Their fundamental mathematical properties are simple. The light reflected from the ground photographed travels in straight lines to the camera lens, and thence to the photographic negative. A pencil of light rays reaching the lens from any point on the ground is brought back to convergence or focus in the corresponding image at the focal distance behind the lens. For all practical purposes, there is a cone with its vertex in the lens, one nappe cut by the negative, one by the ground. Each

* The expression, "conic projection," herein signifies a projection by rays constituting a cone, a sense supported by natural meaning and certain usage, which should be distinguished from use of the same expression to refer to any projection to a conical surface.

element of the cone represents the axis of a pencil of light rays from ground point to image point. If a plane be passed through the optical axis, the light rays it contains may be simply represented by straight lines crossing at the lens as in the diagrams.

Scale and Area of Single Photograph.—Fig. 1 represents the ideal case—a horizontal photographic plate and level ground. A lens with a focal length, f, projects light rays from the points, A and B, on the ground to the image points, a and b, on the photographic plate. From the similar triangles, $A\,B : a\,b :: H : f$, and the photographic scale, $S = \frac{A\,B}{a\,b} = \frac{H}{f}$; that is, the scale of the photograph is the ratio of the height of the lens to the focal length. If, as is customary, H is expressed in feet and f in inches, then $\frac{H}{f}$ is the scale in feet to the inch; if expressed in the reverse ratio, as is frequently done in the case of maps, such ratio or "principal fraction" is $\frac{f}{H}$.

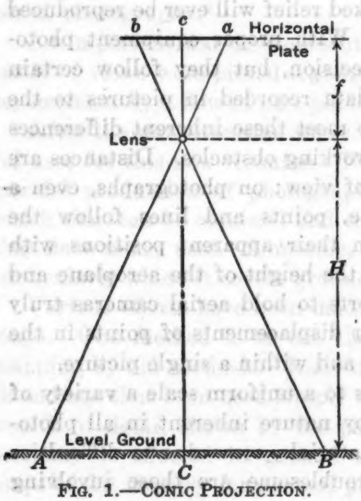

FIG. 1.—CONIC PROJECTION.

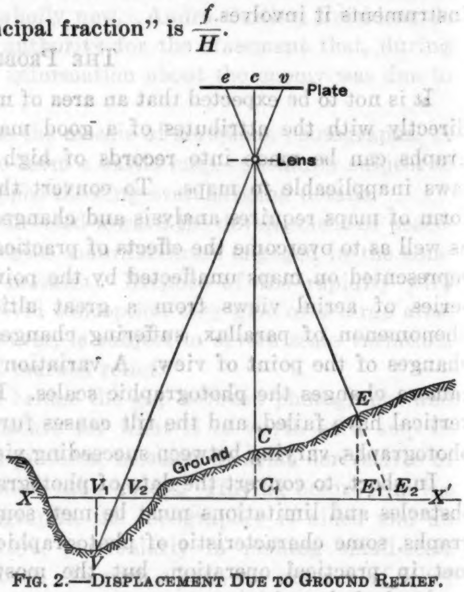

FIG. 2.—DISPLACEMENT DUE TO GROUND RELIEF.

If a and b mark the length or width of the photographic image, from the same proportion, $A\,B = a\,b \times \frac{H}{f}$, which expresses the simple fact that either dimension of the photographic image multiplied by the scale gives the corresponding ground dimension, thus limiting the area that can be shown in the photograph. Similarly, $a\,b = A\,B \times \frac{f}{H}$. By these simple relations it is possible to determine the area of ground represented in a given picture with H and f fixed.

Scale Variation Due to Ground Elevation.—Such ideal conditions seldom if ever exist. If the earth were level, contour maps would have no place. The

scale of an aerial photograph becomes changeable as soon as there is ground relief. Because the photograph is a conic projection, a vertical dimension on the ground causes a horizontal displacement on the plate. Fig. 2 is a simple illustration of this. It represents a section formed by passing a plane through the optical axis and the ground, including a stream channel or valley at V. In an orthographic or map projection to a horizontal plane such as $X X'$, the points, V and E, would show at V_1 and E_1 with their true horizontal distance between, but the conic projection to e and v on the picture plane corresponds to the positions, V_2 and E_2, on the datum plane, $X X'$. That is, to scale, the point, E, is displaced away from the center of the picture by the distance, $E_1 E_2$, and V is displaced toward the center by the distance, $V_1 V_2$, in accordance with the differences in elevation. The projection of the center point, C, is the same regardless of its elevation. If the displacement, such as $E_1 E_2$, be called d, the elevation, such as $E_1 E$, be called h, and the angle, such as $E_1 E E_2$, be called α, then $d = h \tan \alpha$. As $\tan \alpha$ varies with the distance from the axis of the lens, the displacement varies with this distance and with the ground elevation. At the axis of the lens, α is zero, $\tan \alpha$ is zero, and the displacement is zero, regardless of the elevation, as already noted. In other words, the error in the scale of a photograph due to a difference in the elevation of the ground varies with the slope of the ground, the height of the lens, and the distance of the point involved from the center of the picture. It follows that, due to these causes, there is inherent in the photograph a variation in scale as irregular as the ground relief and varying with the other factors named. "It is not practicable to make accurate use of aerial photographs by scaling."*

In Fig. 2 an arbitrary datum plane, X-X', was assumed. If the photographic scale of any other datum plane be used as a basis, the effect in general is to increase the numerical values for some points and decrease those for others. If the results for uniform slopes be considered and the horizontal plane through the foot-point of the lens be taken as the datum, the total change of scale is then divided between plus and minus amounts for the slopes above and below.

For areas having marked relief these errors are very substantial. Viewed from an altitude of 14 400 ft., over ground having a 3% slope, a point down grade from the foot-point of the lens and 4 500 ft. horizontally from it, is displaced in the photograph inward toward the center of the picture an amount equivalent to 72.5 ft. at the scale of the datum plane through the foot-point. To cite an extreme case, a point up grade on a 30% slope and 5 400 ft. horizontally from the axis of the lens is displaced outward an amount equivalent to 1 492.2 ft.

Such gross errors will exist in a single photograph of bold relief. In a mosaic the errors can be lessened by suitable trimming of overlapping prints, but errors due to relief exist throughout mosaics and maps made from them or from photographs without correction. In fact, this condition makes it impossible to obtain a mosaic with perfect junctions between pictures showing ground relief, because two photographs of the same ground or object, photographed from successive viewpoints, take different forms. Some curious effects

* War Department Training Regulations, Aerial Photographic Mapping, Prepared under Direction of Chief of Engineers, January 23, 1925, p. 9.

of this sort result, as shown in Fig. 3. It exaggerates what happens in the case of a road of uniform grade, running normal to the line of flight. At the bottom of the diagram the road appears in elevation and profile; above are indicated two photographic plates in side view and in plan. The plates are assumed to be horizontal and on the one exposed directly above the road, it properly appears as normal to the course of the plane in its flight, but on Photograph 2, exposed to the right, the road appears at an angle to the flight. In the upper right-hand corner of Fig. 3 is illustrated the impossibility of correctly matching the photographs along the line of the road. The images along the flying line are thrown out of position through the angle marked "displacement" and if the images of one point be made to coincide the other points along the road do not, although on both photographs the road appears as straight. If it is of uniform width on the ground its width on the plate will taper slightly.

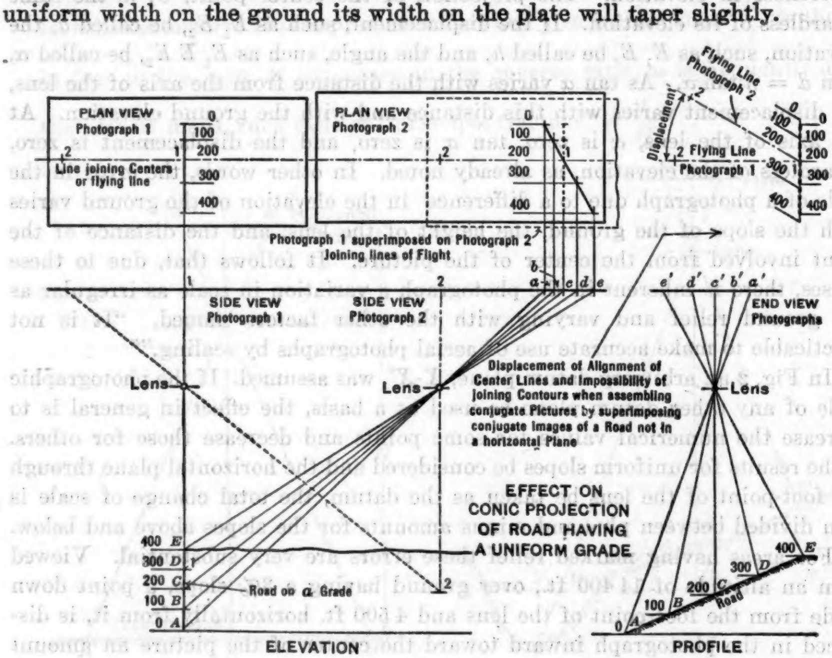

FIG. 3.

Fig. 4 represents a more common case of a road of varying slope. This again may be straight in plan or in orthographic projection on a map, but, on plates exposed in the aeroplane before and after it passes above the road, it will appear as a curve; and the concavities or convexities of the two projections will face each other. At the right the two are superimposed, showing the impossibility of trimming prints so as to make them join along such a line.

Ground relief, then, causes the displacement of points in the photographs, and, for a given elevation, the displacement in a single picture differs for points of different distance from the axis of the lens. Similarly, for the same point on the ground a change in the position of the lens and the optical axis causes a change in the displacement of the point in the photographic images.

Thus, in two photographs taken with the axis of the lens vertical but on opposite sides of the same point on the ground, it is displaced in opposite directions, that is, away from the axis of each position or the center of each picture. In other words, in overlapping photographs images common to both suffer a change of relative position, or parallax.

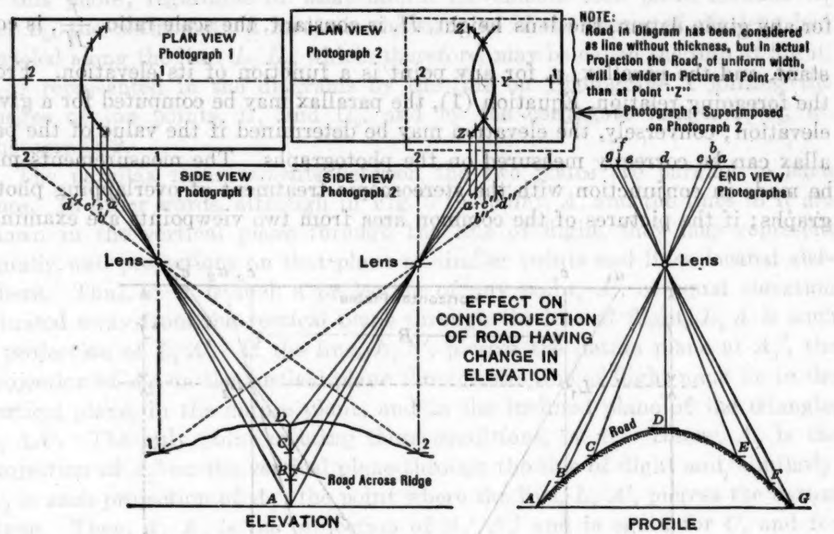

FIG. 4.

Parallax.—Parallax in general is an apparent change of position of objects due to a change in the point of view. Familiar examples of this are the relative shifting of objects seen through the window of a moving train and the different readings of time by persons looking at a clock from different angles. Similarly, in overlapping aerial photographs, points in relief are projected differently on a datum plane. Thus, Fig. 5 represents two plates exposed in the same horizontal plane above the points, C_1 and C_2, on the ground with the lens at successive points, L_1 and L_2; on the first plate the point, A, on the ground has its image at a_1, corresponding to A_1 in the datum plane through C_1, whereas on the second plate its image is a_2, corresponding to A_2 in the datum plane. The horizontal distance between C_1 and A is represented by $a_1\,c_1$ and $a_2\,c_1'$ on the two plates, corresponding to the lengths, $A_1\,C_1$ and $A_2\,C_1$, in the datum plane. In the photographic scale of the datum plane the difference in the length of the two images is: $A_1\,C_1 - A_2\,C_1 = A_1\,A_2$. The triangles, $A_1\,A A_2$ and $L_1\,A L_2$, are similar, and, $A_1\,A_2 : L_1\,L_2 :: h : H - h$. Then,

$$A_1\,A_2 = L_1\,L_2 \frac{h}{H - h}$$

if the distance between exposures, or the base, $L_1\,L_2$, be called B, the difference of distances, $A_1\,A_2$, be called D, and the corresponding difference in length

of image, or parallax, be called p, then p represents D to the photographic scale of the datum plane, that is,

$$p = \frac{f}{H} \times D = \frac{f}{H} \times B \frac{h}{H - h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For any pair of plates exposed in a horizontal plane, the base, B, is constant, for any given datum, the lens height, H, is constant, the scale ratio, $\frac{f}{H}$, is constant, and the parallax, p, for any point is a function of its elevation. From the foregoing relation, Equation (1), the parallax may be computed for a given elevation; conversely, the elevation may be determined if the value of the parallax can be correctly measured on the photographs. The measurements may be made in conjunction with the stereoscopic treatment of overlapping photographs; if the pictures of the common area from two viewpoints are examined

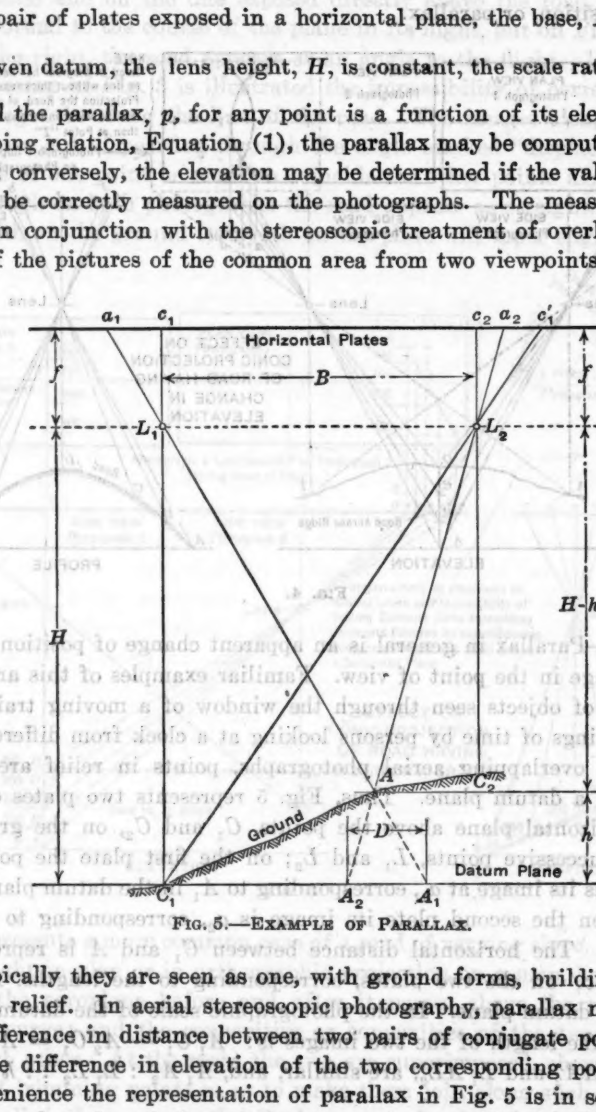

FIG. 5.—EXAMPLE OF PARALLAX.

stereoscopically they are seen as one, with ground forms, buildings, and other objects in relief. In aerial stereoscopic photography, parallax may be defined as the difference in distance between two pairs of conjugate points (images) due to the difference in elevation of the two corresponding points (objects). For convenience the representation of parallax in Fig. 5 is in some respects a special case, but the relations deduced may be generalized. Thus, the point, A, is shown as above the datum plane, but might have been shown below, in which case, $A_2\ C_1$ would exceed $A_1\ C_1$, the difference, $A_1\ C_1 - A_2\ C_1$, would be negative, and for a minus elevation the value of the parallax would be negative, as it should be. For a point in the datum plane the elevation would

be zero, and likewise the parallax, for the difference, $A_1 C_1 - A_2 C_1$, would be zero.

Again, for simplicity Fig. 5 shows relations in a single plane; it represents a section through the two vertical positions of the axis of the lens. The effective movement of the aeroplane and camera between the two exposures has been in this plane; regardless of what lateral movements took place between L_1 and L_2, the mathematical result is the same as if the center of the lens had traveled along the line, $L_1 L_2$, which, therefore, may be called the line of flight. It is represented in the diagrams by the line on Photograph 1 joining the images of the points, C_1 and C_2, and by the conjugate image, $c_1' c_2$, on Photograph 2.

The parallax measurements between the two plates are parallel to such lines. In other words, although in Fig. 5 the point, A, and the lines to it are shown in the vertical plane through the line of flight, they may represent equally well projections on that plane of similar points and lines located elsewhere. Thus, if A is such a projection of any point, A', of equal elevation situated away from the vertical plane through the line of flight, $L_1 A$ is such a projection of $L_1 A'$. If the line, $L_1 A'$, pierces the datum plane at A_1', the projection of A_1' on the vertical plane through the line of flight must lie in the vertical plane, in the datum plane, and in the inclined plane of the triangle, $L_1 AA'$. The only point meeting these conditions, is A_1. Hence, A_1 is the projection of A_1' on the vertical plane through the line of flight and, similarly, A_2 is such projection of A_2', the point where the line, $L_2 A'$, pierces the datum plane. Then, $A_1 A_2$ is the projection of $A_1' A_2'$ and is again for C, and for any point having the elevation of A, the measure of "the difference in distance between two pairs of conjugate image points due to the difference in elevation of the corresponding object points", provided these distances and difference are measured parallel to the line of flight. That is, parallax values as here defined are always to be measured parallel to the line of flight for a pair of photographic plates.

Limitations of Photographic Material

This is not the place for a discussion of all the minutiæ of the photographic art. Only a few points are mentioned that assume special importance in connection with the use of photographic materials for producing engineering maps.

One requirement is that the base carrying the negative emulsion shall be flat or so nearly a perfect plane as not to cause errors in highly refined measurements. It is strange that in all that has been written regarding aerial photography so little has been said concerning this feature which is so fundamental and which, if in error, is quite impossible of correction.

Film.—Precise measurements have disclosed errors due to the use of film, which make it inappropriate for the purpose of aerial mapping. Various expedients, such as pressure plates and suction in different forms, have been tried with the same results. Films are distorted by the developing process, which in many instances frees the gelatine emulsion locally from its celluloid base in patches, adding further errors to the general shrinkage that takes place.

Even without separation or "creep", the shrinkage is uneven in different directions. To quote M. André Carlier again, "However, we must state that at present for all work requiring great precision it is necessary to use photographic plates".* M. Carlier's conclusion is the more significant because he might be expected to have a partiality for film due to his experience with it in the war, when he was in charge of aerial photography at the front.

Plates.—The emulsion does not creep on glass plates, nor do such plates shrink in development. Ordinary photographic plates, however, may have considerable curvature, often as much as 15′ of arc. No means is available to correct the effect of this in a single photograph or to avoid its cumulative effect in a series. The use of flat plates is, therefore, essential.

Practical Operating Difficulties

The engineer, accustomed to *terra firma* for the legs of his transit or plane-table, expects some practical difficulties in making observations from a vehicle moving through air at a rate of more than 1 mile per min. The difficulty that has proved most troublesome is due to this lack of stability.

Overlap.—Adjoining photographs must overlap. Overlapping is needed if a gap is to be avoided as it is impracticable to take the photographs so as just to join end-on. For the best mosaics, it is needed so as to permit the outer portions of each photograph to be discarded, as already explained. It is necessary in order to orient each picture with reference to those adjoining; for this purpose the two images of the same thing on the two views are utilized. Overlap is also needed so as to permit the stereoscopic treatment of the pictures.

Although overlap is essential for these purposes, it alone does not attain them. Overlap will not eliminate variation of scale nor assure a correct orientation of adjoining photographs.

Tilt.—Ground slope is not the only important cause of the variation of scale in the photographs. A variation is caused in practical operation by the changing inclination of the focal plane. Air pockets, wind changes, banking and tipping of the aeroplane, all tend to rock or tilt the camera. Fig. 6 is a simple illustration of some of the results. It represents a vertical section through the axis of the lens of a tilted plate, and shows also the position the plate would have taken if it had been horizontal over the same position of the lens, that is, the axis of tilt is normal to the plane of the diagram through the common position, *L,* of the lens. The trace or intersection of the horizontal and tilted planes of projection is likewise a line normal to the diagram through the point, *m.* For convenience of illustration, the angle of tilt as shown is excessive, but this does not affect the principles involved.

For the horizontal position of the plate, the optical axis is vertical, intercepting the ground at *V,* whereas, for the tilted position, the optical axis is swung through the angle of tilt and intercepts the ground at a very different point, *C;* that is, the centers of the two negatives will receive images of different ground points and the distance and direction between them will vary with the varying degree and direction of the tilt.

* "Cependant, nous devons dire qu'à l'heure actuelle pour tous les travaux exigeant une grande précision, il faut se servir de la plaque photographique," *L'Illustration,* 13 December 1924, p. 585.

The areas photographed will differ accordingly. Some light rays, such as *K L*, reaching the horizontal plate will not reach the tilted plate; some rays, such as *A L*, will reach the tilted plate, but not the horizontal plate; some ground dimensions, such as *M J*, will have a greater image on the tilted plate than on the horizontal; and the opposite will be true of some dimensions, such as *M B*. The trace of the horizontal and tilted planes of projection receives the same light rays in either case and the photographic scale in that one line is unaffected.

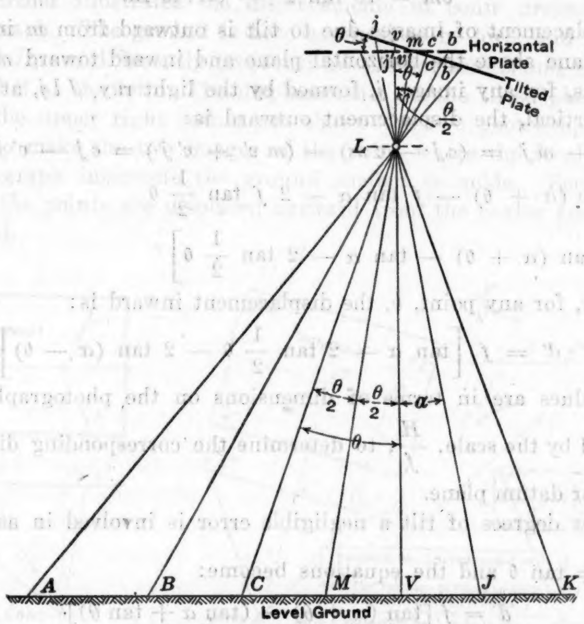

FIG. 6.—DISPLACEMENT DUE TO TILT.

The axis of the lens for the horizontal position pierces the plate in its optical center, v', and the same light ray intersects the tilted plate at v. The axis of the lens for the tilted position pierces the plate in its optical center, c, and the same ray cuts the horizontal plate at c'. The angle of tilt is:

$$\theta = v\ m\ v' = c\ m\ c' = v\ L\ c$$

The focal length, $f = L\ v' = L\ c$, and hence the right triangles, $L\ v'\ c'$ and Lcv, are equal; that is, $L\ v' = L\ c$ and $v'\ v = cc'$; hence the right triangles, $v'\ mv$ and cmc', are equal and $v'\ m = cm$. Therefore, the right triangles, $Lv'\ m$ and Lcm, are equal, and

$$\text{Angle } m\ L\ v = \text{Angle } m\ L\ c = \frac{1}{2}\ \theta$$

On the tilted plate, v is the image of the point on the ground vertically below the lens, vc is the image of the line connecting the vertical point with the optical center, and m is on that line at a distance from the center,

$$c\ m = c\ L \tan \text{Angle } c\ L\ m = f \tan \frac{1}{2}\ \theta$$

whereas, the distance from the optical center to the vertical point is $cv = f \tan \theta$. For small tilt, such as commonly occurs in practice, $f \tan \theta$ is virtually twice $f \tan \frac{1}{2} \theta$; that is, the point, m, is virtually half-way between v and c and hence it is sometimes called the mid-point. Thus, for 2° of tilt, with a focal length of 10 in.,

$$cv = 10 \tan 2° = 0.3492 \text{ in.}$$
$$cm = 10 \tan 1° = 0.1746 \text{ in.}$$

The displacement of images due to tilt is outward from m in the part of the tilted plane above the horizontal plane and inward toward m in the part below. Thus, for any image, j, formed by the light ray, $J\,l\,j$, at an angle, α, from the vertical, the displacement outward is:

$$d = m\,j - m\,j' = (c\,j - c\,m) - (m\,v' + v'\,j') = c\,j - v'\,j' - 2\,c\,m$$
$$= f \tan (\alpha + \theta) - f \tan \alpha - 2\,f \tan \frac{1}{2}\,\theta$$
$$= f \left[\tan (\alpha + \theta) - \tan \alpha - 2 \tan \frac{1}{2}\,\theta \right]$$

Similarly, for any point, b, the displacement inward is:

$$d' = f \left[\tan \alpha - 2 \tan \frac{1}{2}\,\theta - 2 \tan (\alpha - \theta) \right]$$

These values are in terms of dimensions on the photographs and must be multiplied by the scale, $\frac{H}{f}$, to determine the corresponding dimensions on the ground or datum plane.

For a few degrees of tilt a negligible error is involved in assuming that $2 \tan \frac{1}{2}\,\theta = \tan \theta$ and the equations become:

$$d = f\,[\tan (\alpha + \theta) - (\tan \alpha + \tan \theta)]$$
$$d' = f\,[(\tan \alpha - \tan \theta) - \tan (\alpha - \theta)]$$

In practice the tilt can usually be kept within 2 and need rarely exceed 3 degrees. This, however, causes substantial displacement of the points. Thus, at a height of 12 000 ft., with a lens having a focal length of 12 in., producing a scale of 1 in. = 1 000 ft., in the case of a point so located above the mid-point that $\alpha = 25°$, a tilt of 2° would cause a horizontal displacement equivalent to a ground distance,

$$D = H\,[\tan 27° - (\tan 25° + \tan 2°)] = 99.5 \text{ ft.}$$

Serious errors also result in determinations of ground elevations from tilted plates. Thus, if the plate last defined were used for parallax readings in conjunction with a horizontal mate, with an exposure interval of 3 000 ft., or a base of 3 in. on the plate, the indicated elevation of the point, B, would be 20 ft. too high, that of the point, J, 17.5 ft. too low, or a total error of 37.5 ft. in the difference in elevation between the two points.

From the equations it will be noted that for a given scale the displacement in terms of ground measurements is proportional to the focal length. Thus, the device of flying at a greater elevation and increasing the focal length

proportionally so as to retain the same photographic scale and reduce the displacement of points that is inherent in the conic projection of ground elevations, has the disadvantage of increasing proportionally the displacements due to the tilt. A photographic mosaic from original negatives will be subject to variations in scale from both causes (ground relief and tilt); for a given photographic scale any change in flying height and focal length will decrease the error from one cause and increase that from the other.

Fig. 7 further illustrates the displacements in conic projection on the tilted photograph. The ground is here assumed to be level and four points, *A, B, C,* and *D,* are differently projected on the horizontal and tilted Photographs 1 and 2 at the left. The displacements on the tilted photograph are radial. At the upper right corner are shown the two photographs superimposed so as to make the two images of the point where the focal axis of the second photograph intersects the ground surface coincide. Because of the tilt some of the points are displaced outward from the center and others are moved inward.

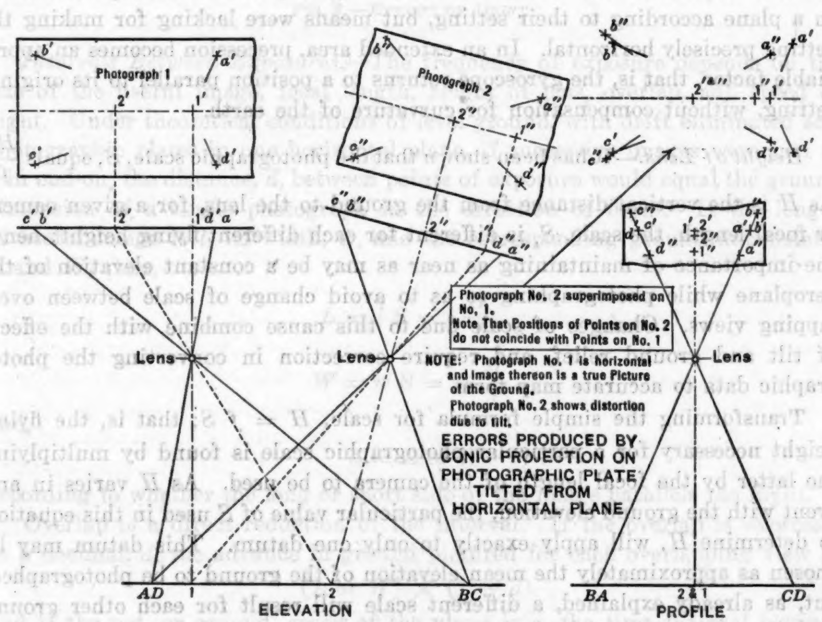

FIG. 7.

Variation in scale due to tilt alone is illustrated in Figs. 6 and 7, but the common condition is that this is combined with displacements due to elevation and other factors in uncertain and varying amounts. These diagrams show the effects in a single plane or direction; like effects occur in other planes and directions. Thus, the results in the photograph are complex. The addition of tilt to the other causes of changes in scale is a complication that has caused much difficulty to those attempting the translation of the photographic data into maps and has been the undoing of many such efforts.

Earnest endeavors have been made to prevent tilt—to avoid at the source the variation in scale from this cause. All such experiments with which the writer is familiar have served at best only to minimize it. The camera may be suspended above its center of gravity so that any tilt from the vertical immediately enlists the force of gravity to bring the camera back, but such action will not prevent slight tilting this way and that at the instant of exposure. Another simple device that suggests itself is to attach spirit-levels to the camera and to exercise positive control over its position in accordance with their readings; or to photograph the bubbles with the picture as a record of tilt and as a guide for its correction. The difficulty of this device is that the bubbles themselves are seriously affected by the accelerations and decelerations of the aeroplane. Their readings are useless for accurate work.

Gyroscopes have been used, but in the extended experiments with which the writer is familiar they have proved insufficient. Suspended pendulously, they tended to keep the camera vertical, but the corrective action was too slow to preclude tilt. Suspended neutrally, they tended to keep the camera in a plane according to their setting, but means were lacking for making the setting precisely horizontal. In an extended area, precession becomes an appreciable factor, that is, the gyroscope returns to a position parallel to its original setting, without compensation for curvature of the earth.

Height of Lens.—It has been shown that the photographic scale, S, equals $\frac{H}{f}$. As H is the vertical distance from the ground to the lens, for a given camera or focal length, the scale, S, is different for each different flying height; hence the importance of maintaining as near as may be a constant elevation of the aeroplane while photographing so as to avoid change of scale between overlapping views. Changes of scale due to this cause combine with the effects of tilt and ground relief, and require correction in converting the photographic data to accurate map form.

Transforming the simple formula for scale, $H = f\ S$; that is, the flying height necessary for a particular photographic scale is found by multiplying the latter by the focal length of the camera to be used. As H varies in any event with the ground elevation, the particular value of S used in this equation to determine H, will apply exactly to only one datum. This datum may be chosen as approximately the mean elevation of the ground to be photographed, but, as already explained, a different scale will result for each other ground elevation appearing in the individual picture or mosaic.

Speed and Drift.—The speed of a plane in still air is partly under the control of the pilot, through his control of the motor and its fuel. Speed is also affected by the velocity of the wind with respect to the line of flight. If the flight is with the wind, the speed of the plane is greater than it would be in still air. If the directions of flight and of wind are opposed, obviously the speed of the plane is reduced. Under working conditions, the net or "ground speed" will ordinarily vary between about 60 and 120 miles per hour, or roughly between 90 and 180 ft. per sec. If the wind crosses the line of

flight, the plane must be pointed so that the resultant of the forces carries it in the desired direction.

Ordinarily, flights in a strong wind will not be made, but to compensate for any cross-wind the plane must be pointed at a slight angle from the line of flight and the camera should in turn be swung back through the same angle; otherwise, a strip of pictures presents a serrated border, as indicated in Fig. 8. It is not imperative that this effect be eliminated, but for convenience and economy it should be minimized.

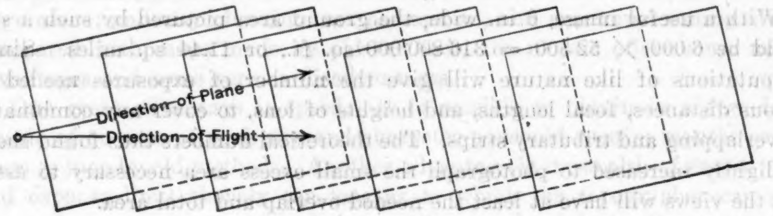

FIG. 8.—EFFECT OF DRIFT.

Intervals Between Exposures.—The frequency of exposure depends on the size of the useful image, focal length, height of lens, overlap, and speed of flight. Under theoretical conditions of level ground, with drift eliminated and photographic plates in one horizontal plane, if successive images were just to join end-on, the distance, d, between points of exposure would equal the ground dimension of a single photograph in the direction of flight. If the length of useful image is l, its width, w, and the corresponding ground dimensions, L and W, then,

$$L = l\,S = l\,\frac{H}{f},$$

$$W = w\,S = w\,\frac{H}{f},$$

and,

$$d = L \text{ or } W$$

according to whether the long or short side of the image parallels the flight.

Overlap is a direct reduction of the interval. If the overlap is expressed as a decimal, d, the advance in ground pictured for each overlapping view is,

$$(L \text{ or } W) \times (1 - d)$$

and if the net, or ground, speed of the plane is g, the time interval between overlapping exposures is,

$$t = \frac{(L \text{ or } W) \times (1 - d)}{g}$$

Thus, for a speed of 60 miles per hour, or 88 ft. per sec., at an elevation of 12 000 ft., with a 12-in. lens and an image 8 in. in the direction of flight, for a 60% overlap:

$$t = \frac{8 \times 12\,000 \times 0.4}{12 \times 88} = 36\tfrac{1}{2} \text{ sec., approximately.}$$

Number and Area of Overlapping Exposures.—For a strip of pictures extending for a distance, D, on the ground the number of plates or exposures is,

$$N = 1 + \frac{D - (L \text{ or } W)}{(L \text{ or } W) \times (1 - d)}$$

Thus, for a flight as in the preceding example for a distance of 10 miles, or 52 800 ft.,

$$N = 1 + \frac{52\,800 - 8\,000}{8\,000 \times 0.4} = 15.$$

With a useful image, 6 in. wide, the ground area pictured by such a strip would be $6\,000 \times 52\,800 = 316\,800\,000$ sq. ft., or 11.44 sq. miles. Simple computations of like nature will give the number of exposures needed for various distances, focal lengths, and heights of lens, to cover any combination of overlapping and tributary strips. The theoretical numbers thus found should be slightly increased to photograph the small excess area necessary to assure that the views will have at least the needed overlap and total area.

Optical and Mechanical Requirements

Instruments of precision are the foundation of modern surveying. The engineer of to-day takes his transit and level for granted, yet all his work of triangulation and precise leveling, his comparisons of stadia and plane-table, his other elections in methods of control, mapping, and location, rest on that little group of instruments that exercise so profound an influence on his work. The development of successful designs of the engineer's transit and other instruments demanded years of painstaking effort.

Aeroplane topographic surveys call for new instruments of precision. They are as essential to the new art as the transit and level to ground surveying. The design and construction of such new instruments, to equal or exceed the old in their accuracy and to meet the requirements of precision and practical workability, offered a problem demanding similar effort and skill. Under pressure of other demands, the aeroplane has attained sufficient development and variety so that it may be taken for granted. It must be supplemented by instruments for accurate aerial photography, for the determination and removal of tilt, for the interpretation and conversion of the conic projection in photographs to the orthographic projection and contour delineation in topographic maps.

The camera should be at least as accurate as the transit. As film is to be avoided, a plate camera is needed. To permit sufficient exposures on a single flight, magazines for the plates are required. The instrument used to "horizontalize" the plates must be essentially a photographic projector, with provisions for tilting the negative and copy board. The instrument for interpretation of the photographs must include provision for refined measurements of the displacements already mentioned, so as to determine the effects of tilt and change of elevation. In all the instruments, the lenses should be as free as possible from distortion, the different elements should be truly concentric, and, in general, the optical devices should be of high precision in design, assem-

bly, and adjustment. A partial description of the instruments that have been designed and used for aeroplane topographic mapping in connection with the method under consideration, are here given.

Mosaics

Reference has been made to the mosaic as a development of the World War. Such a composite picture has many peace-time uses, including the preliminary consideration of rights of way for electrical transmission, railroads, and other lines, general studies of drainage basins, timber holdings, and other large areas; and city planning, tax assessment, and municipal zoning studies. Compared with maps made by old methods, the mosaic offers some marked advantages and some serious disadvantages.

One great advantage is speed. It is possible to obtain in a few days a mosaic of an area of such size and character as would require months or even years to map by old methods. Another advantage is its wealth of detail. Time and expense limit sharply the lengths to which the topographer can go in measuring stream meanders, timber outlines, and other details; conventional signs limit what the cartographer can show; but the camera gives a picture of the whole. While the transitman locates a point or line, the camera records a square mile.

The weakness of a mosaic, fatal to its use as a substitute for the engineering map, is its lack of uniform scale. It offers little or no indication of elevations. Its component parts are subject throughout to irregular changes of horizontal scale due to ground relief and tilt, as already noted. The assembly of these is an aggregation of the local errors, and offers opportunity for additional mistakes in orienting the fragments and fitting them together. A straight line on the mosaic will usually represent a broken or curved line on the ground and *vice versa*. Thus, the mosaic has been called "a caricature of the landscape."

An obvious method of merging overlapping prints into mosaic form is to superimpose conjugate images of two or more sharply defined points or features, tear or cut through the overlap, and waste the excess fragments, continuing the process with adjoining pictures to the limits of the area. Errors of orientation result from differences of elevation. Thus, if one point used is in the datum plane and the other a high point, the result is as if the second print had been rotated about the first point and out of correct position an amount dependent on the elevation or resulting parallax of the second point.

Of the scale variations or errors in mosaics, those due to tilt can be eliminated if the tilted negatives can be so re-projected as to be "horizontalized" before taking prints for making into mosaics, but the errors, usually greater, due to ground relief, remain. The latter, being inherent in photographs, are inevitable in mosaics. They can, however, be lessened by shortening the interval between exposures, thus increasing the number of photographs and restricting the fragments used to smaller areas about the center points of the pictures. They can be lessened and localized if the centers of the pictures can be correctly located by platting and the fragments oriented; but the difficulty has been to identify precisely conjugate images of the center points. In

any event none of these devices supplies a real map because of the lack of vertical scale and the inevitable variation in the horizontal scale. If the centers of the pictures are correctly platted to a chosen scale and correctly oriented, the photographs of ground between center points will have varying scales for different elevations of ground, and in trimming adjoining prints some of the ground will not be shown or will be duplicated in the mosaic.

A Solution

The first step in the method under discussion is to secure a series of overlapping aerial photographs on glass plates. Suitable pairs of plates are examined stereoscopically, and the tilt is determined and overcome by re-projection. Contours are drawn on the plates and all data are converted to orthographic projection, with such constant scale as is desired on the final tracing. By means of these successive steps there is produced from aerial photographs an accurate contour map in the usual form of tracings and blue prints as long used by engineers. The instruments and technique used have been developed through continuous research since the conclusion of the World War.

Field Operations

Flying.—Aeroplanes of various types are used, according to the requirements of flying radius, ceiling, and weight-carrying capacity. Ordinarily, a field base is established near the area under survey and thus the flights are not sufficient to tax these requirements. A two-man crew is used, pilot and photographer. The area to be mapped is traversed in straight lines representing photographic strips, with overlapping strips and tributary lines as needed to cover the area with a simple network of nearly straight lines.

The effort is to fly on an even keel at a constant height, usually approximating 5 000, 10 000, or 15 000 ft. If it is necessary to determine the best program for covering the area under survey, a preliminary reconnaissance flight is made for that purpose.

Photographing.—Overlapping views are taken with an interval between exposures fixed so that each photograph will include the center point of any adjoining picture. Because of the inadequacy of film and of the curvature likely to exist in ordinary photographic plates, only selected flat glass plates are used. To secure a record of sufficient accuracy and extent without excessive flying, a magazine camera of special design is used. Its details and construction are such as to handle glass plates easily and rapidly, producing overlapping photographic views of such character as can be utilized in the special apparatus necessary for the later processes.

Promptly after each photographic flight, the plates are developed and the prints are made and examined to assure that the desired area and overlap have been secured. In case of uncertainty regarding the exact area to be covered by the survey, a preliminary mosaic is used to define this area.

Ground Control.—To determine the effect of the tilt of the camera and interpret the data recorded in the photographs some ground measurements are made, but they are merely a small fraction of the work that would be involved in a ground survey for comparable results. For horizontal control, an isolated

base line, preferably about 3 000 ft. in length, is measured at a convenient location near each end of a network of photographs, and every 15 or 20 miles in a long series. For vertical control, differences of elevation are measured on the ground for several points that appear on each plate; the distances between these points are not needed.

Points used for horizontal and vertical control must be identified clearly not only on the ground, but in the photographs. Sharply defined points are selected from field prints of the photographs by use of a hand stereoscope. Although there is considerable latitude in the location of the points, the effort is to secure a point near the center of each picture and one near each end of a line through the center normal to the line of flight. For an overlap exceeding one-half the picture, this means that nine such points appear on any particular photograph within a strip, and that six of the points may serve also for adjoining photographs as indicated ideally in Fig. 9. Thus, the number of field points per square mile depends on the size and shape of the area as well as the photographic scale. As the scale, S, equals $\frac{H}{f}$, an increase of the flying height or a decrease of the focal length lessens the number of field points required per square mile. For a flight at an elevation of 10 000 ft. with a lens of 8 in. focal length and a 6 by 8-in. image for three parallel strips of ten plates each, and an overlap of 60%, there would be 50 field points in about 22 sq. miles, or about 2 per square mile.

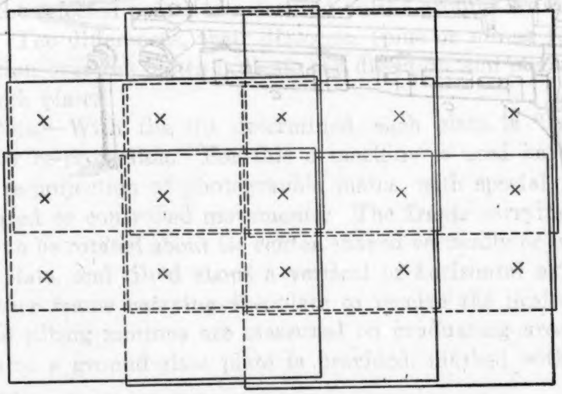

FIG. 9.—FIELD POINTS FOR OVERLAPPING PLATES.

Office Operations

Centering Negatives.—The optical center of each plate (negative) is permanently marked. This operation takes little time and consists of simple details, but deserves description because of the effect on accuracy. The plate is laid on a glass table and a straight-edge is laid diagonally across it so as to join the fine marks it carries as a result of pin-holes in the aperture plate of the camera. A short mark along each diagonal is cut in the emulsion making an intersection that marks the center of the picture. The accuracy is enhanced by use of needle points, razor edge, and magnifying glass.

From this negative plate a positive is made by direct projection and used in subsequent operations. The glass positives carry all the attributes and markings of the negatives, including the center points.

Stereoscopic Examination.—The measuring stereoscope, shown in Fig. 10, consists essentially of systems of lenses for the stereoscopic examination of photographic plates mounted on turn-tables, with provisions for passing light through the plates from below and for certain measured movements in the plane of the turn-tables. These movements include the rotation of each table about its center, the movement of one table along the line joining their centers, and the joint movement of both along that line and normal to it.

In Fig. 10 the lens housing and eye-pieces (*a*) appear at the top and a pair of plates (*b*) appear on the turn-tables below. Hand-wheels control the various motions. Lying in racks against the front of the machine may be seen two rules (*c*) made of glass with metal frames. The glass parts contain horizontal and vertical cross-hairs carefully set in the long and short axes of the rules.

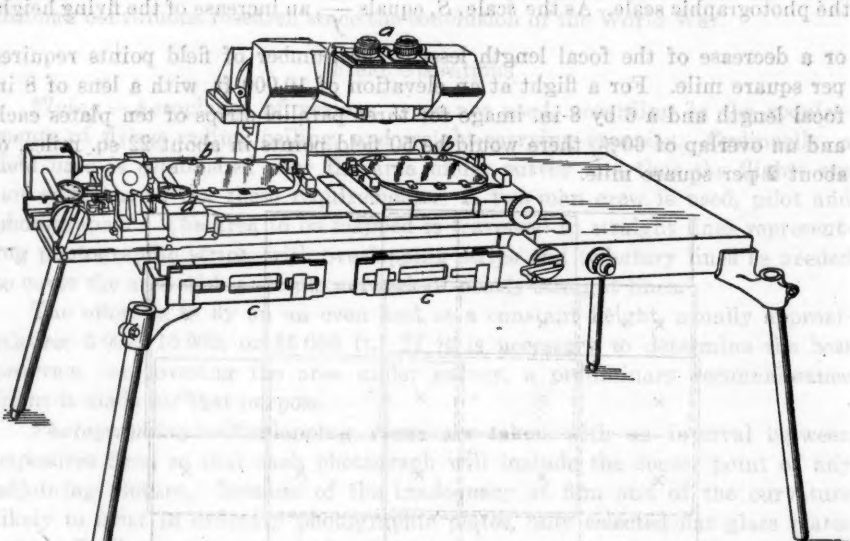

FIG. 10.—MEASURING STEREOSCOPE.

The rules fit recesses provided for them on the turn-tables and the intersecting cross-hairs of the rule then mark the mechanical centers of the tables. These are used to bring such centers separately in line with the optical axes of the stereoscope. This adjustment once made need not be repeated unless the machine is disturbed for some reason.

The two positive plates under examination are placed on the turn-tables, each separately centered under the cross-hair intersection of the eye-piece and

clamped. This adjustment once made need not be repeated until a new pair of plates is mounted on the tables.

The tables are rotated so as to bring into alignment the four points on the plates marking the two views of the center point of each plate. This involves identification on each plate of the conjugate center or image of the center of the other plate. This can readily be approximated by observing the picture point with the naked eye, and can be accurately done by stereoscopic examination. This alignment once being established, the motion of rotation is not again used until a new pair of plates is mounted on the tables.

The center of one plate and the conjugate image on the other plate are separately centered under the eye-pieces. The bed carrying both turn-tables is then moved laterally (along the line of flight) until on the first plate the conjugate image for the center of the other plate is centered under the eye-piece. The other eye-piece will then not be over the center of the other plate. In other words, due to parallax and other causes the two images of the line between centers of the plates will usually be of different length. The micrometer dial (*d*) measuring the amount of divergence between the plates is set at zero. The other plate is moved independently until it is centered under the eye-piece. The micrometer reading will then show the "spread." Ordinarily, this will not be due solely to the difference in elevation (parallax), but will involve also the factor of tilt in one or both plates.

Similar parallax readings are taken for the few salient points previously mentioned. The correct (computed) parallax values based on the known elevations of the selected points as measured on the ground are compared with the readings. The differences, their direction (plus or minus readings), and their distribution over the plates indicate the direction and amount of the tilt in either or both plates.

Re-Projection.—With the tilt determined, each plate is "horizontalized" if necessary by re-projection. For this a machine is used having the usual facilities for re-projection of photographic plates, with special provisions for certain measured or controlled movements. The frame carrying the plate to be projected can be rotated about its center, moved vertically or laterally in the plane of the plate, and tilted about a vertical or horizontal axis through its center. Another frame carrying the plate to receive the projection has like motions. The tilting motions are measured on graduating arcs. For use in the latter frame a ground-glass plate is provided, marked with vertical and horizontal axes.

The plate to be projected is put in position and its center is brought precisely into line with the focal axis of the lens. It is then rotated about its center until the conjugate image of the center of the other plate is brought into the horizontal plane through the center and focal axis. The tilting motions are then manipulated in the direction and degree indicated by the differences between parallaxes already computed from known elevations and those indicated by the preliminary stereoscopic examination. This adjustment is divided between the negative and the ground-glass plate so as to keep a sharp focus throughout the view, and then the ground-glass plate is removed, a sensitive photographic plate is substituted, and the re-projection made. The plate

thus secured will approximate very closely the result that would have been obtained had the original aerial exposure been made on a horizontal plate.

The various plates needing correction are similarly re-projected and the accuracy of the results is verified by re-examining the plates in pairs on the stereoscope, comparing anew the parallax readings with the known parallaxes for the salient points. In rare cases a plate is re-projected a second time for greater accuracy. On the other hand, an original plate is at times such a close approximation to the horizontal as not to require re-projection at all. Thus, finally a complete series of overlapping views on horizontal planes are obtained which are used in subsequent steps of the process.

Contouring.—Two plates representing overlapping views are mounted on the stereoscope, centered and aligned as previously described. Starting with a low point of known elevation, the spread between the plates is set to correspond with the computed parallax for the elevation of the nearest contour line desired. The plates are then moved together and observed stereoscopically until a point is located under the centers of both eye-pieces, that is, a point having such a parallax. This is a point on the desired contour and is marked with a special pencil on one plate. Sufficient other points are similarly located to build up the contour line on the plate. The computed parallax for the desired contour interval is added to the reading of the machine and the next contour line is similarly located and drawn. This process is continued until the area common to the two plates is fully contoured on one of them. The plates are then removed and the pencilled contour lines are drawn through the emulsion with a needle-point.

This process is greatly facilitated by the stereoscopic vision of the pictures. The cross-hairs seem to lie on the ground surface when a correct point is brought under their intersection. In case of a point too high the cross-hairs seem to bury themselves in the earth, and in the case of one too low they seem to float in the air above it. With this characteristic and the vivid picture of relief seen through the stereoscope, the skilled operator in practice can quickly and accurately trace any contour line. Fig. 11 shows a plate contoured throughout. In practice the contours are drawn on one view of each area common to an overlapping pair. The process is continued with the plates in pairs until by the aggregation of such parts the entire area is contoured.

Plotting and Tracing.—After contouring, the product is a set of plates representing the conical projection of the desired area on horizontal planes with at least 50% overlap in adjoining plates and contour lines of desired elevations and interval. As a contour line connects images of equal elevation, it marks a level plane and its conical projection on the horizontal plate has uniform scale. The scale varies, however, as between the different contours, but the scale of any contour line can be modified as desired by moving each point of the image radially.

To control the drawing, the centers of the pictures and other selected points are plotted separately. For selecting the control points the photographic plates are examined in pairs on an illuminated glass table and common images, sharply defined and located toward the edges of the plates, are

FIG. 11.—CONTOURED PLATE, GREEN RIVER SURVEY.

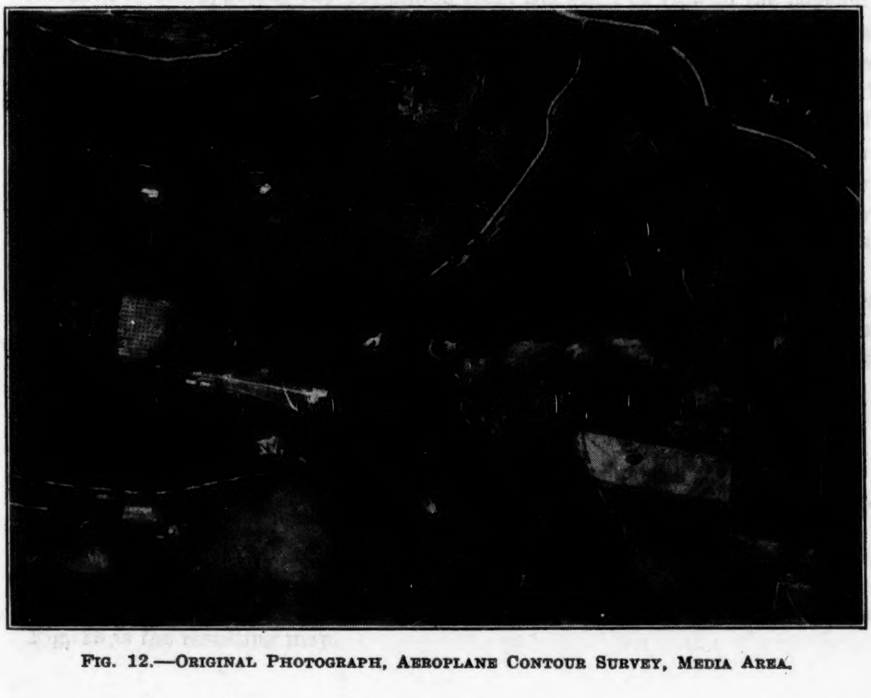

FIG. 12.—ORIGINAL PHOTOGRAPH, AEROPLANE CONTOUR SURVEY, MEDIA AREA.

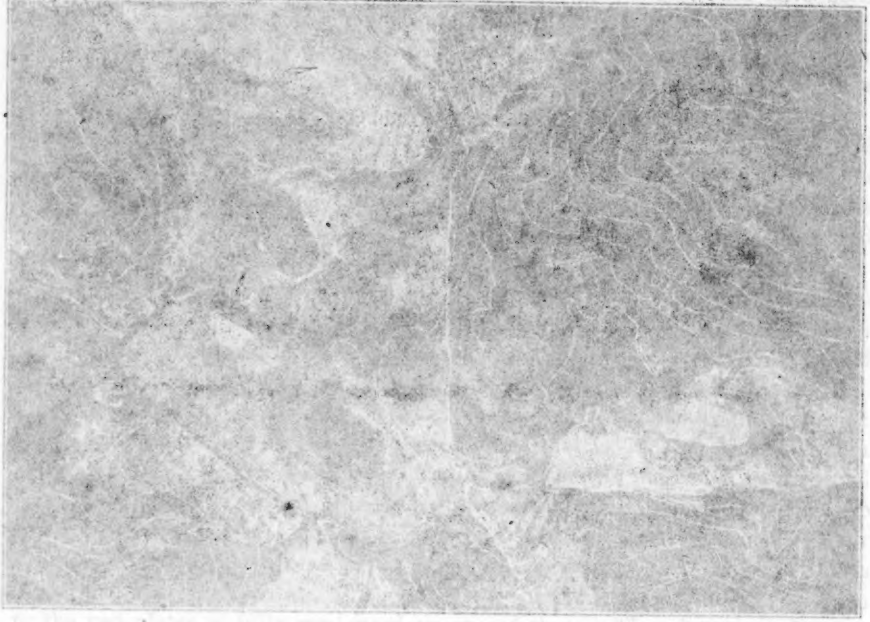

FIG. 11.—CONTOURED PLATE, GREEN RIVER SURVEY.

FIG. 12.—ORIGINAL PHOTOGRAPH, AEROPLANE CONTOUR SURVEY, MESA AREA.

chosen. At least six points are marked on each plate, usually more. For the plates containing the base line measured on the ground the plotting points include the ends of this line. The plotting of the map commences with the base line. Its known length is plotted with great care on drawing paper to the scale desired on the map. The centers of plates including the base line are then located with reference to it by means of direction lines taken from each plate. These "ray" lines from the centers of the plates to the ends of the base line and to other plotting points are correct in azimuth regardless of the changes in elevation or scale, provided the plates are "horizontalized" (corrected to the horizontal), just as horizontal angles or azimuths read by the transit are correct regardless of differences in elevation, provided the transit is properly adjusted to the horizontal. Similarly, the intersections of such ray lines from the different plate centers serve progressively to orient the succeeding plates and plot the various selected points. In this way, the points chosen on each plate are successively plotted, and, finally, the contour lines, drawn on the plates as already described, are readily transferred to the finished map with such corrections in scale as are necessary to make the final drawing correct to scale throughout.

Results

Aeroplane contour surveys are new, but have been subjected to some rigid tests. The originators of this method first sought an independent review of its correctness theoretically. A number of investigations and reports were obtained by them, giving expert opinion from the point of view of the physicist and the engineer, all of which were entirely favorable. Practical work was then undertaken, and a number of surveys have been, and are being, carried out. In all instances the results have been very satisfactory and the parties for whom the maps were made, or their engineers, have made highly favorable reports. The maps have been compared critically with independent ground surveys made by transit and other instruments at construction sites or more generally over the areas involved. In one instance the aeroplane surveys disclosed a substantial error in the ground surveys, but in no case has the reverse occurred.

In 1921, the writer was assigned with other engineers to make a careful examination of the theories involved and to check the results obtained. One of the aeroplane contour surveys studied was for a district near Media, Pa., having the following characteristics:

Location: Delaware County, Pennsylvania.
Approximate area mapped: 10 sq. miles.
Character: Moderate slopes, small streams, much culture.
Approximate elevations: Maximum, 300; minimum, 87; difference, 213 ft.
Horizontal scale: 1:6952;
Contour interval: 20 ft.

Fig. 12 is one of the original photographs made in the Media survey and Fig. 13 is the resulting map.

Accuracy

The Media survey, made in 1921, was of an experimental nature, and was carefully checked by the writer. For this purpose, simultaneously with the photographic survey, a "stadia traverse," or transit survey, was made using the ordinary instruments and methods. This was not a complete topographic survey and no contouring was done, but the traverse followed generally the principal roads, locating also prominent landmarks, and closed or checked on its starting point with unusual accuracy, so that it offered a satisfactory ground control or reference line with which to verify the photographic results.

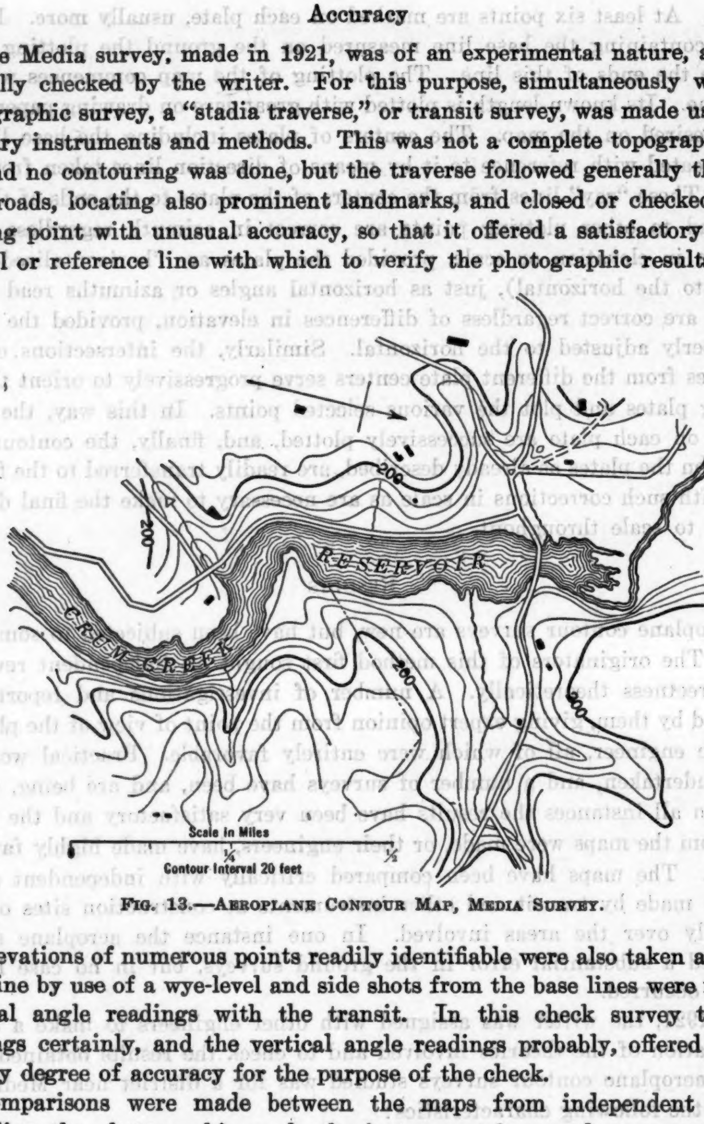

FIG. 13.—AEROPLANE CONTOUR MAP, MEDIA SURVEY.

Elevations of numerous points readily identifiable were also taken along the base line by use of a wye-level and side shots from the base lines were made by vertical angle readings with the transit. In this check survey the level readings certainly, and the vertical angle readings probably, offered a satisfactory degree of accuracy for the purpose of the check.

Comparisons were made between the maps from independent sources, including the photographic work, the instrumental ground survey, and such other maps as were available. Study and comparison of these disclosed the fact that, to a highly satisfactory degree, the aeroplane photographic map coincided throughout with the instrumental survey. Such displacements as were contained in the photographic map were uniformly distributed and non-cumulative, gradually melting out over long distances, so that the relative positions of adjacent points were everywhere correct.

In the 12-mile circuit of the ground traverse, elevations were determined for about 400 points and compared with those shown by the aeroplane contours.

In the subsequent report the conclusion was reached that aeroplane contours could be located with the assurance that none would be in error more than $2\frac{1}{2}$ ft. The following is quoted from the report* dated June 4, 1924: "The accuracy with which this can be done with mechanical limitations of the present machines is a plus or minus $2\frac{1}{2}$ ft."

In discussing the use of the stereoscope, the report continues:

"The same results could be secured by different men familiar with the operation of the stereoscope, proving that the operation was a mechanical one not influenced by the introduction of the human element."

Important improvements subsequently added to this process of making contour maps from aeroplane photography, should further increase the degree of accuracy. The theoretical considerations would point to an accuracy within 1 ft., plus or minus. This obviously is a very high degree of exactness for topographic maps of large areas.

Detailed checks were likewise made of two other surveys. One of these was along the Susquehanna River, near Conowingo, Md., and the other in the Green River Basin of the Blue Ridge Mountains in North Carolina. The topography elevations, scales, and other details differed, but the results disclosed the same degree of accuracy. The wide differences in character of terrain among the three cases offered an unusual opportunity for checking the accuracy and applicability of the method to different types of topography.

Time

One of the advantages of aeroplane contour surveys is the great saving in the time required to produce maps as compared to older instrumental surveys. In the original field observations the aeroplane and camera are so much more rapid than the transit, level, and plane-table as to preclude comparison. The plate correction, contouring, and other office operations connected with aeroplane surveys are also relatively rapid and are susceptible of partition to an extent that makes for maximum speed.

This advantage in time is particularly marked for a large area of bold relief and forest cover, as illustrated in the Green River survey. To quote H. R. Faison, Assoc. M. Am. Soc. C. E., the Resident Engineer who used the maps on that survey:†

"Maps can be completed in progressive sections, as desired, and are in the hands of the engineers for study in a fraction of the time required for ground surveys."

Costs

Unit costs of aeroplane topographic surveys vary with numerous factors, among which the following deserve special mention:

(*a*) Proximity of suitable landing ground and other facilities.
(*b*) Situation of the area, affecting the cost of flying or of transporting the aeroplane to the vicinity;
(*c*) Size of the area, over which such costs must be pro-rated;

* Report by Day & Zimmermann, Inc., Engineers, Philadelphia.
† *Engineering News-Record*, March 5, 1925.

(*d*) Shape of the area, affecting the number of flying hours and photographs;
(*e*) Character of the area affecting the labor required to depict the relief and other features;
(*f*) Contour interval; and
(*g*) Horizontal scale, affecting both field and office processes.

Obviously, these elements cover a wide range, within which costs vary with other factors, including the choice of equipment (notably planes), and the focal lengths of cameras. Detailed cost data have not been published, but are gradually being accumulated and indicate advantages for the aerial method, particularly in topographic surveys of large areas having bold relief and considerable growth of timber or brush. Under such circumstances the data available to the writer indicate that aeroplane topographic surveys may be made for a fraction of the expense required for comparable results by instrumental surveys on the ground. In connection with many projects the greatest savings that can be effected by aeroplane topography are the indirect ones of quicker returns and less interest on waiting capital by virtue of the great saving in time and reduction of the non-productive period devoted to preliminary investigation and survey.

Field of Application

Aeroplane topographic surveys have a broad field of usefulness. The resulting contour maps offer a superior basis for studies of water supply, power, irrigation, and other engineering projects. The unusual combination of speed, accuracy, and detail make the method ideal for use in comparing alternative plans for determining the best plan of development. The maps serve for preliminary location or project layout, for determining catchment areas, for capacity curves for reservoir sites, and for many other purposes for which topographic maps of engineering accuracy and suitable scales are used.

Accurate maps can be produced by this method with horizontal scales such as 200, 300, 400, 600, or 1 200 ft. to the inch. Contours with intervals of 5, 10, or 20 ft., or more, can be located accurately. For more general purposes maps can be produced with horizontal scales of 1 in. to ½ mile, or more, and such contour intervals as 50 or 100 ft.

The advantages of this method are greatest in country of rugged topography and rough surface, but any type of ground can be mapped by this means. The instruments developed serve with the same facility to reduce the photographic information to true-scale maps in the case of flat, rolling, or mountainous terrain. Simple topographic shapes and the complex forms of erosion and glacial action are mapped with the same ease. Even a substantial growth of timber or brush does not defeat the process. The instruments magnify the vertical dimension and even where the ground is quite obscured in the perspective view of a single photograph, it may be clearly visible throughout the stereoscopic view. The contours of the ground surface are uniformly determined.

Origin and Personnel

In this discussion of aeroplane topographic surveys the attempt has been made to state some important elements of the problem which any effort at a solution must meet. Necessarily it sets forth the particular method with which the writer is familiar, that developed by the engineers at the Arthur Brock, Jr., Tool and Manufacturing Works, Incorporated, which is fully covered by patents issued and pending, both as to the method and the special apparatus invented for its convenient practice. As far as the writer knows, it is the only method devised in the United States for the development of contours directly from the data recorded in aerial photographs, and the one such method of any origin to date that is accurate and practical. The method is being utilized under the general supervision of F. E. Weymouth, M. Am. Soc. C. E.

HISTORY AND PROBLEMS OF IRRIGATION DEVELOPMENT IN THE WEST*

By John A. Widtsoe,† Esq.

The Civil Engineer has long held irrigation leadership in the world. It is fitting, therefore, that the program for a meeting of the Society held in the West should concern itself with irrigation matters.

Periods of Irrigation History

Within the Great West of the present United States, there are three periods of irrigation history—the prehistoric, the pre-pioneer, and the modern. Considerable information is available concerning each of these periods.

The Prehistoric Period.—Irrigation was practiced in the West before modern man took possession of America. Remnants of irrigation canals and structures in California, Arizona, New Mexico, Colorado, and Utah, bear witness to a dense population that throve under the irrigation ditch.

The Pre-Pioneer Period.—The pre-pioneer period represents the meager attempts at irrigation by those who came into the Great West as trappers, traders, ranchers, and missionaries before 1847. The Roman Catholic Fathers established missions in Arizona, New Mexico, and California, around which were built small irrigation systems. Occasionally, members of the great body of fur trappers and traders would divert water from a stream in the great unsettled West and raise a crop or more of grain or vegetables. The Spanish Government undertook, about 1796, to build an irrigation community in California, near the present site of Santa Cruz, under the direction of Lieutenant Alberto de Cordoba. Most of the principles at present advocated for the settlement of irrigated lands were used, but the venture ultimately failed. Until 1847 all Western irrigation ventures by white men were small and not on a permanent community scale.

The Modern Period.—The modern period began on July 24, 1847, when a company of Mormon pioneers, led by Brigham Young, entered the Great Salt Lake Valley, and on that day spread water from City Creek over land which was being plowed. From this initial experiment have grown the vast irrigation enterprises of Western United States. The modern period is characterized by three classes of enterprises, which succeeded each other, although they frequently overlapped.

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The pioneer enterprises were those in which the farmers themselves, chiefly by their own labor and always with limited capital, in the spirit of co-operation, set about to dig ditches and to bring water on dry land. The pioneer enterprises flourished mostly until about 1880, although, ever since the original Salt Lake Valley experiment, there have been occasional ventures conforming in method to the early enterprises. The pioneer enterprises were not comparable in extent with the irrigation ventures of the present day, but, in view of the difficulties and hardships of the pioneer days, they were remarkable. For example, in the State of Utah, in 1865—18 years after the coming of the first pioneer company—there had been dug, by co-operative effort, 277 irrigation canals of a length of 333 862 rods, costing $1 766 959, and capable of serving 153 949 acres at an average cost of $12 per acre. Besides, in that year, there was in progress the construction of other canals of an estimated cost of $877 730. The Union Colony, the forerunner of the great irrigated section centering about Greeley, Colo., is another famous example of a pioneer irrigation enterprise.

The prosperity that attended these pioneer enterprises was so marked that capital soon became interested. The capitalistic enterprises then followed the pioneer enterprises. It seemed as if the investment of capital in irrigation enterprises would be safe and profitable. Vast areas of land were brought under irrigation by the capitalistic enterprises. In fact, in 1900, 9 000 000 acres of land in Western America had been brought under cultivation, at a cost of from $15 to $20 per acre. It was discovered, however, that many difficulties and dangers attend an irrigation enterprise that is not principally fostered, supported, and managed by the water users themselves. It seemed to be demonstrated that, ordinarily, a capitalistic irrigation enterprise is not profitable. Finally, it became clear that the development of the irrigated section was lagging because the precarious financial returns of irrigation investments made capital reluctant to engage in irrigation enterprises. Meanwhile, it was fully admitted that the development of irrigation was highly beneficial to the country.

In 1902, Congress passed and President Roosevelt signed the famous Reclamation Act. Thus, the Federal-aided irrigation enterprises were inaugurated. This unique instrument for the conquest of the arid and semi-arid lands of the Republic by irrigation, provided that the proceeds from the sales of public lands in the Western States, later augmented by royalties from oil and other lands, were to be used for the building of irrigation works. The farmer was to pay no interest on the investment, but should return the cost in easy installments. Thus, a perpetual revolving fund would be created which, in time, would utilize all available water on the lands of the western part of the United States.

The United States Reclamation Service, in the two decades following the passage of the Reclamation Act, expended nearly $150 000 000 in the construction of magnificent irrigation works of outstanding merit, brought about 2 000 000 acres of land under the ditch, and contributed a volume of experience to irrigation practice, before unknown in the history of the world. The annual crop income from the Federal irrigation projects was upward of $75 000 000.

The passage of this Act, and the consequent activity in the construction of irrigation works, had the further effect of stimulating the interest of capital in irrigation ventures. From 1902 onward, therefore, there was another great increase in the irrigated area. In 1920, the irrigated section had expanded to 20 000 000 acres, at a cost of $30 to $50 per acre, and the crops harvested from the irrigated section in that year were valued at about $760 000 000. Numberless cities, towns, and villages covered the West as a result of this tremendous irrigation enterprise. Roads and railroads intersected the irrigated section and all the institutions and organizations of modern civilization had arisen on the reclaimed deserts of the West.

It would seem that, then, with the experience gained by the pioneer, the capitalistic, and the Federal-aided enterprises, the principles of irrigation would be so well understood as to insure the permanence and prosperity of the practice. Yet, at this very time, irrigation was, so to speak, placed on trial before the people of the United States—and the hearing is not yet ended. The older methods of founding and conducting irrigation enterprises appeared to be insufficient under the changed economic conditions of the day. Seepage and other insidious dangers had developed which threatened the permanence of the movement. The humid area called attention to its own lands not under the plow. Abnormal post-war conditions raised the specter of over-production. Colonization of the irrigation projects was slowed up. The newer projects carried an increasing acre cost, often beyond the reach of the water user. The water users on the Federal irrigation projects were not repaying the construction costs, in spite of the fact that several relief measures had been passed making it easier for the farmers to repay. It seemed at first that the Reclamation Service alone was on trial, but it soon developed that the whole irrigated area was under consideration by those who questioned the soundness of the movement. Many of those who raised questions relative to the future of irrigation were friends of the cause, who desired to save reclamation by irrigation, as they thought, from destruction. Others were frankly enemies to Western development. During the last two years investigations have been made by the Federal Departments and laws have been enacted by Congress which seem to promise that whatever changes are necessary to make the movement stable and successful will be made. Meanwhile, a new day in irrigation is before the people of the West. There must be a revision of principles and methods to conform to present needs.

The Problems of Irrigation

Irrigation is a complex art, involving the application of the best knowledge of agriculture, especially the relationships among soils, crops, and water; of engineering, especially of hydraulics; of rural economics and sociology; and of the life and response of human beings under the unique conditions always prevailing under the irrigation ditch.

It is not surprising that so complex an art, at this time, when thoroughgoing changes are reshaping economic and social structures, should present many and serious problems. Such problems pertaining to the development of irrigation in the West do exist, and must be solved, if the great American

experiment in irrigation shall become permanently successful. Yet these problems, essential to irrigation development, may be classified very simply into two groups:

1.—To establish on every irrigation project an acceptable economic and social environment.

2.—To extend gradually, as economic needs demand, the irrigated area by completing existing projects and by reclaiming new areas, until the water resources of the West shall be fully utilized.

An Acceptable Economic and Social Environment

The heroic figure in irrigation development is the water user—the man and woman who throughout the years live upon the land, till it, and make it yield enough for life's sustenance. Capitalist and engineer, statesman and tradesman, are all necessary, but compared with the water user they are of secondary rank in bringing irrigation development to success as measured by the standards of the age. The water user is as the great river that drives the wheels of the power plant; all other workers in behalf of the project are as the contributing side streams, higher up, that swell the volume of the river.

The tremendous importance of the water user in successful irrigation development has not been understood clearly; less important, although essential, factors, have been stressed. The result has been, too often, distress or failure on corporate and public irrigation projects. Experience has given its order, "right about face"—face to face with the needs of the water user. This is the large modern problem in the development of irrigation in the West.

Agriculture under any clime—humid, arid, or irrigated—will always remain a mode of living, a manner of life, rather than a business in which much money is made. Only those who are content with the normal rewards of rural life will succeed and be happy on the farm. Yet the dawn of the new agriculture gives promise that, as a result of his labors, the farmer may participate in the joys and satisfactions of progressive civilization. His income must suffice to provide the things that the majority of men find desirable; that is, a prime concern of irrigation development will be to strike a proper balance between the annual income and the obligations of the water user. The income must come from the lands; and the obligations must include all costs of a modest, thrifty, but comprehensive life in this day of the latest achievement; that is, irrigation farming must be made profitable. Any irrigation enterprise which does not make this thought fundamental is foredoomed to failure. Colonization, social development, and all other present needs of the irrigated West, depend on a readjustment by which the farmer can meet his obligations from his land income and yet live life as men should live it.

This problem applies to all farming, but its solution varies with environment. The irrigation farmer lives under the ditch. He has a control over water not possessed by his brother in the humid regions. He is not dependent on the rainfall. Herein lies a power which, properly used, may free him from useless competition and make his products desired by all markets. In addition, he lives under a clear sky. Climate and water re-act on crop production and composition. The peculiarities of irrigation environment must be cap-

italized to bring about full irrigation success. This phase of the problem is not beyond the field of the engineer. The trained mind is needed, however the training has been secured. The problem is worthy of the best thought.

Then, if the capitalist, whether private or governmental, will refuse to construct or support any irrigation project which cannot enable the water user to "make a living", the highway to the solution of this problem will be reached. Successful water users make successful projects. How to make water users successful is the first problem.

The Extension of the Irrigated Area

The East has need of the West; and the West has need of the East. The country must be developed as a whole, and not in sections. The activities necessary for the development of the vast resources of the West will cluster about, and depend on, the irrigated centers. The venture in irrigation will not be wholly satisfactory until the irrigated area has been extended to the full utilization of the water resources of the West. This cannot be done at once; nor should it be done hastily. Yet, if the work is to go on properly, there must be a constant movement toward the full realization of irrigation possibilities.

The Completion of Existing Projects

Few irrigation projects in the West are completed to meet present needs and possibilities. The newer of the existing projects are not yet well-seasoned, many of them not fully colonized, and few of them under an acceptable program of agriculture. They need much help before they can be viewed as established ventures.

The early projects were small, the water supply was relatively large, and the engineering devices were simple. As the population grew, the water was made to cover more land than was originally intended. As this spread became wider and the depth of water thinner, the danger of the dry year became more evident, until to-day there is a demand on most of the older projects for more water in the critical months and in the dry years. This condition requires that reservoirs be built to hold back the flood waters of normal years to supplement the present supply, and to store the excess of the wet years to serve as a hold-over supply for insurance against the dry year.

The older projects developed gradually. The first canal was followed by later canals that merely extended the original project. The result is that frequently the higher canals irrigate the lower-lying lands. On some projects this situation, the gradual result of irrigation development, is chaotic and expensive. There is need on many of the older projects of a complete re-organization of canal management, having in view lower operation costs, greater return flow, and a higher economy in water use.

Wherever water falls there is a power possibility. There is an increasing need of power on the farm. Irrigation by pumping is assuming a higher importance. The commercial demand for power is becoming larger. Power production on irrigation projects cannot be ignored. Provision must be made for power development, by the well-planned use of the water in streams and

canals devoted to irrigation purposes, even if the power generation occur only during the irrigation season. Underground waters will be needed in irrigation development, and summer power will find large service in the use of such water supplies.

These and many other problems, such as that of full colonization, in connection with the completion of existing irrigation projects must be given early attention. There are hundreds of small private projects which, with a little help of the kind mentioned, would greatly increase in efficiency. We have been very prone of late years to value irrigation ventures according to size. The small venture is usually the most successful.

Irrigation Extension by Building New Projects

Although the completion of the older projects, small and large, should be the first concern in extending the irrigated area, yet the reclamation of lands remote from existing projects, for which water may be secured, must not be forgotten. In the main, such future projects require the construction of expensive and difficult irrigation works and Federal aid, with interest-free money and easy terms of repayment, is indispensable. Difficult as these structures may be, high as the price may be, it will be found profitable to bring such lands under irrigation. In time, the country will need all its irrigation resources. Meanwhile, such projects should be undertaken slowly, gradually, as economic need is foreseen, and in accordance with a definite plan covering the whole West. There must be no haphazard irrigation construction if one intends to win success. The program of extending the irrigated area by building new projects should be preceded by an irrigation survey and stock-taking. Legislators and Courts must be willing to revise irrigation codes, on the basis of modern knowledge, for a full, efficient, and economical development.

The Means for Solving Irrigation Problems

Request has not been made for suggestions as to the solution of the irrigation problems. However, to mention some of the means at hand for such solution may not be improper.

First.—There must be established, within the Nation, a new faith in irrigation. It must be shown that the development of the arid and semi-arid section is a National need. The people must be made to understand that the vast resources of the West—mining, forestry, ranching, and dry-farming—can be made really successful only if they are in association with irrigated centers and with a wise program for the full, although gradual, development of its irrigation possibilities.

Second.—A new comprehensive study must be made of the total irrigation needs and possibilities of the West. This would include a consideration of each irrigation project. It would be an irrigation stock-taking of the United States. Clear thinking would result from such knowledge.

Third.—The great treasures of irrigation knowledge won during the last few decades in research laboratories, on experimental farms, and from practical experience, should be organized and used more freely. Within a genera-

tion, materials for a science of irrigation have been gathered. Unfortunately, applications of this knowledge have not kept pace with the material gathered.

Fourth.—Reclamation by irrigation must be approached hereafter with a new appreciation of the need of economic and social emphasis. There has been a tendency to trust that, when water and land are brought together, successful irrigation enterprises will be brought forth. It is now known that this is not so. Economic and social principles as well as physical and biological laws must be brought into operation to make irrigation projects successful. Many or most of the disasters which have befallen irrigation in recent years appear to have been due to the failure to recognize these social and economic factors.

Fifth.—Training must be provided for the irrigation leaders who shall build upon the magnificent foundations laid by the present and earlier leaders. Such training must be made to conform with the best knowledge and views of the present day. Engineering schools may well accept the responsibility for such training, even to the extent of a partial re-organization of the traditional engineering courses.

For some years there has been a growing feeling that the standard college course in engineering is so highly technical and of so narrow a range that the graduate is not able to make the best application of his training in a world of men and women, touching many and varied activities of life. Certainly, those who in the future shall take and hold leadership in the irrigation development of the West, must know, in addition to the fundamentals of engineering, some agriculture, economics, and law. The engineering schools which desire to serve in the irrigation development of the West, may be obliged to formulate a new course of instruction based on the irrigation knowledge and experience gained during the last two decades.

Conclusion

It has been the speaker's good fortune during 1924-25 to spend nearly half his time in an exhaustive study of the irrigation conditions of the West, especially as pertaining to the Federal reclamation projects. He has come out of the work with renewed confidence in the value of irrigation to the West and to the country as a whole. The difficulties that have arisen, the failures that have been recorded, are not arguments against irrigation, but rather pointers to the way which we must tread in the future. Without question, the future of American irrigation is secure if the means at hand are used in the solution of the relatively few problems connected with the development of irrigation in the West.

THE FINANCING OF IRRIGATION DEVELOPMENTS BY PRIVATE CAPITAL*

By R. E. Shepherd,† Esq.

Synopsis

The substance and the conclusions of this paper are largely the result of experience in solving the problems of the Twin Falls North Side Project. This is a privately financed enterprise. The failure of the original promoters forced the property on the bondholders, for whom the speaker has been engaged for the past eleven years.

The Twin Falls North Side Project is one of the large projects of the West, covering 185 000 acres of irrigated land. In the course of this work nearly, if not quite all, the problems peculiar to irrigation have been encountered. Among them has been the question of a dependable water supply for the abnormal year.

The question of an adequate water supply not only affected the "North Side", but to a more or less degree most of the projects in the Snake River Valley of Idaho. The solution lay in building a large storage reservoir at American Falls on Snake River. This was a joint enterprise, participated in by many, including the U. S. Reclamation Service. To secure the required funds, the North Side and other near-by projects, covering about 450 000 acres of irrigated land, organized themselves into a quasi-municipal district, known as the American Falls Reservoir District. This district entered into a contract with the United States for the building of the reservoir, and for about 300 000 acre-ft. of storage capacity therein. It issued its bonds for about $2 000 000 to cover the contract and paid the money to the Government. This arrangement has proved very satisfactory.

It will be noted that the lands were already under irrigation and that no new canal construction was required. The whole object was to remedy a defective water supply. For this reason the bonds of the District—6%—sold readily at par and a small premium, the country served being already generally settled and productive.

Many other problems have presented themselves for solution in the course of the speaker's experience on the North Side; these, and his observations of other projects, have led to the conclusions contained in this paper.

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Dependence on agriculture is recognized, as well as the fact that growth in population requires a constant increase in production of food and those raw materials which must come from the soil.

Practically all the public domain lying in the humid area suitable for agricultural use in its natural state has passed into private ownership and is now to a large extent devoted to farming. It is true that much of it is capable of increased production, the necessity of which has for some time engaged public attention. There are limits fixed by cost which for the present define the extent to which the ever-growing need of the country may be supplied from this source.

It has been found that the arid lands of many parts of the West possess wonderful fertility, and where irrigation was practicable large areas have been devoted to agricultural production. It would be interesting to inquire into the results already secured, the wealth produced, and the influence on the Nation as a result of the present reclamation of the arid lands of the West. It supports large systems of railroads, makes a market for manufactured products, supplies much of certain raw material, and contributes a large part of the food supply of the United States. Therefore, it may be taken for granted that the reclamation of arid lands is something to be desired, and that it should be done at such times and in such areas as will fit in with the changing needs of the country.

Recognizing all this, engineers are here concerned with the question of how shall irrigation development be financed. Shall it be by private capital or out of public funds? To what extent is it prudent for the Nation to embark in any business enterprise? Should the Government participate in, and co-operate with, private enterprise and capital in the work of land reclamation?

Where the land to be reclaimed is in private ownership, and where there is a near-by source of water supply and the problems of storage reservoirs and interstate streams are not involved, clearly such work should be left to the enterprise of the owners of such lands. Practically all the earlier reclamation work related to such conditions was undertaken by those directly interested and was financed by private capital. The growth of such work led to the enactment of laws by most of the States, providing for the creation of irrigation districts of a quasi-municipal character, for the construction, operation, and financing of irrigation systems to serve the lands within such districts. In this manner provision was made for the reclamation of larger bodies of land in diverse ownership, but having a common need of irrigation and capable of being served by one irrigation system and from the same source of water supply.

Under this system, which has been gradually perfected by legal enactments as time and experience have shown the necessity, most of the work of reclamation under private enterprise and capital has been accomplished. The results under this system have not been uniformly successful, and much loss of capital and effort has resulted from lack of proper knowledge of the real merits of the project, or its true cost. The losses under this system, however, were not inherent in or peculiar to the system, but were due to the same

preventable blunders and ignorance which produce failures in all misconceived enterprises.

Can private capital be reasonably assured of safety in any irrigation enterprise? There are certain questions that must be satisfactorily answered before any project should be undertaken. It seems strange to one looking back over the failures in this kind of work to find how little attention was paid or study given to the work undertaken, a most casual investigation of which often would have prevented inevitable loss, both to the investor and to the trusting farmer.

The order in which these questions will be presented does not signify their relative importance, as the speaker believes that if any project will not at the time stand up under all of them it should be let alone until such time as it will. Some of these questions go to the very vitals of an irrigation project, whereas others relate to the time of the undertaking. Changed conditions may make desirable in the future that which would now be unprofitable. Satisfactory answers should be required and proof offered as to each of the following questions. If doubt exists as to any one, time should be taken to clear it up. Practically all the problems, formerly more or less hidden, have come to light in the disasters of existing irrigation projects, so that knowledge and information, once guessed at, are now available.

1.—Is the land to be reclaimed sufficiently fertile and its soil structure such as to produce valuable crops for an indefinite period without resort to excessive cost for fertilization?

2.—Is the surface of the land such as to permit of its irrigation without too great expense, having due regard to the class of major crops for which it is adapted?

3.—Can the land be readily drained at reasonable cost, so as to prevent its becoming swampy or alkalied, after repeated irrigation?

4.—Is there an adequate supply of water available within a reasonable distance sufficient at all times to supply all the land in the project with the quantity required to produce profitable crops?

5.—Can this water be diverted for this purpose through a canal system that can not only be built within reasonable cost, but can thereafter be operated and maintained without excessive expense?

6.—Is the project so located with reference to transportation and markets as to offer an incentive for its farm development?

7.—Is there a present market demand for the products for which such land is naturally adapted, that under ordinary conditions would make the use of such land profitable to a farmer of ordinary ability and means?

8.—Will the entire cost of the work, including the time required to secure settlement on the land, plus a reasonable profit, when spread over the entire area, be such that the acre cost to the farmer, including the original cost of the land and its improvement by him, compare favorably with the cost of a farm in the humid sections, having due regard to character of crops, yield, and cost of production?

9.—Is the project so financed to the farmer that it will attract the man of limited means to locate thereon with assurance that he can succeed and meet his obligations promptly?

There are other questions that may enter into each particular project not common to all, but satisfactory answers to each of the foregoing should be made before venturing on the enterprise, and when these are had, then capital may safely undertake such development.

The speaker wishes particularly to dwell on a few of the fundamentals. The question of water supply is one of the most common errors and has caused more loss than all the others combined. The discharge of a given water-shed is not uniform. There are years of high stream flow and years of low; many factors enter into it. The information on a particular stream may be unreliable. Then, again, there may be prior rights on the stream not fully understood or fully developed, which may later reduce the supposed right of the project. Too great care cannot be exercised in obtaining accurate information on this subject. Many old projects, which were assumed to have good water rights, are now finding it necessary to build storage reservoirs in order that flood waters may be accumulated for later use.

Then, again, experience has shown that no man can with exactness foretell what the transmission losses in a canal system will be. So much of the water diverted from the stream may be lost in transmission, that the quantity reaching the farm may prove inadequate, requiring the area of the project to be reduced with the attending loss of investment. Peculiarities are likely to develop in the canal system, which could not be reasonably anticipated, limiting the farm deliveries. The question of farm duty of water cannot well be accurately predetermined.

On the other hand, it is known that as a farm is developed and reaches a stage of good cultivation, as the humus content in the soil increases by vegetation, as the farmer himself acquires greater knowledge in the use of water on his land (which can only come with time), and as a greater diversification of crops is grown on the project, less water is required to overcome losses.

Therefore, the speaker has long since reached the conclusion that in some respects an irrigation project is more or less a case of evolution. It should be undertaken in units wherever possible, having always in mind that successful farming is the end and aim of irrigation development. No irrigation system should, or can, carry its maximum load during its early years; it will grow, but the exact limit of its growth cannot be known at the start. It, therefore, is good judgment to finance irrigation development with a due regard for these inherent characteristics.

Another class of irrigation projects and their relation to private capital and enterprise, and some of their problems will now be discussed. In most if not all the Western States, there are large bodies of land suitable for farms, but involving the use of so much capital and the solving of so many new problems that private capital and enterprise hesitate to undertake their reclamation. Without taking the time to review the history leading up to the pas-

sage by Congress of the Reclamation Act, the Carey Act, and the Warren Act, a brief statement of the reclamation work undertaken in consequence of these acts and some of the results may prove helpful in the present discussion.

Under the Reclamation Act the Government undertook, through a bureau created for the purpose and now known as the Bureau of Reclamation, to construct irrigation works for the reclamation of certain parts of the public domain in the arid West. The funds for this purpose came from certain revenues of the General Land Office and have since been increased from other sources. The details were left largely to the discretion of the Secretary of the Interior. The reclaimed lands were made subject to entry, conditioned that the entryman should assume and pay the cost of construction, to be thereafter ascertained and published by the Secretary of the Interior, as well as the annual cost of operation and maintenance. Wide publicity was given to this new field of reclamation work. Each State was active in securing projects, and a general invitation was extended to take up farms thereon. The applicants were many, and a system of drawings not unlike a lottery was provided.

The first noticeable result was that a large part of the lands were drawn by those not expecting to make a so-called farm home. Then came the realignment. As the construction cost was to be paid in small installments, without interest, commencing after the cost had been ascertained and announced, and as there was a general expectation that this cost would be much less than that of a private enterprise, entrymen were able to, and many did, sell at good profit to those unlucky at the drawing. So the unearned increment was guessed at and quickly capitalized, and the final holder of the land got it at about the going price of other lands, and subject to, in many instances, an unascertained construction cost.

All this, however, is aside from the real work of reclamation undertaken by the new Bureau, and much of the criticism growing out of it should rest on other shoulders. Great care was exercised in the study of each project, and in the water supply therefor. The work in most, if not all, these projects was well done and a credit to those in charge. However, the cost frequently exceeded that expected by the entryman, or subsequent purchaser, and is now the subject of much controversy. The usual consequences of land speculation induced by the World War are now in evidence; whether this could have been prevented does not enter into the question being considered. It will be noted, in passing, that if the Government does finally absorb some of these losses, it must do so for other reasons than that the lands and irrigation works acquired from the Government were not worth the charge.

The real question is, "Was the work worth what it has cost?" There may be some projects where it is not. If so, the speaker is not familiar with them. It should cause no surprise if in a work so extensive, covering such great diversity of conditions, there have been some errors of judgment.

At about the time of the Reclamation Act, another known as the Carey Act was passed, having the same general object in view, but under which the work was to be undertaken by State supervision. By this Act any State having arid land of the public domain within its borders capable of irriga-

tion, might have a limited part of it segregated to the State, to be thereafter conveyed by Government patent to the State on a showing that irrigation works had been completed and a water supply obtained for its reclamation. The State, in turn, was to enact suitable legislation to secure this desired result. As part of this program the State was authorized to enter into contracts with such construction companies as might desire to engage in the work of building irrigation systems for the reclamation of certain designated areas thus segregated by the Secretary of the Interior for the purpose. These lands in turn were to be open to entry, conditioned that the entryman must first have purchased a water right for use on his entry from the construction company. The water right consisted of a certain proportionate part of the canal system and water secured for the project. The State, by its contract with the Construction Company, fixed in advance the price per acre to be paid by the entryman for such water right and the terms of payment. The so-called water-right contracts were made a lien upon the land they served, and were by the State contract made assignable to a trustee, as security for an issue of bonds out of the sale of which the funds were to come with which to construct the works. The essential difference, it will be noted, as far as the entryman was concerned, being that under the Carey Act the price was fixed, whereas under the Reclamation Act it was indefinite and left for determination after entry. The result was that many Carey Act projects were started, particularly in Idaho. No end of trouble ensued, as may well be imagined. It was something new. Many contracts were loosely drawn, and endless litigation finally resulted.

At the start the State used every endeavor to induce capital to embark in these enterprises. Engineers readily pointed out the great opportunities. Certain bond houses saw a chance at profitable financing, and soon, without the careful study which has been pointed out, numerous projects were started and people rushed to them as to a new gold field. These Carey Act projects have been more or less of a disappointment and loss to many.

The root trouble in most of these projects grew largely out of two causes: First, under-estimating the construction cost, which fixed in advance the sale price of water rights; and, second, over-estimating the available water supply. These companies were held to their contracts on the question of price and the Courts required the areas to be reduced to fit the water supply, where it could not be improved.

This was the first big venture of the general public in irrigation investment. It proves nothing for or against the broad question of the legitimate development of arid land by irrigation, but it does lead to the inevitable conclusion that left entirely to the direction of either State or Nation it has not so far been a financial success.

Growing out of the experience of the work of the Reclamation Bureau and the Carey Act companies, and the fact that they were often both looking to the same source of water supply, came the Warren Act, which permitted of joint undertaking in building reservoirs. This Act was followed by others encouraging co-operation between all irrigation districts and companies with the U. S. Bureau of Reclamation in the work of conserving the waters of

the great water-sheds, and in the correction of many of the original mistakes. A notable example of such co-operation may be found in the building of the American Falls Reservoir in Snake River, now under construction. The past mistakes are gradually being corrected and thousands of acres of fertile land are now being successfully farmed as the result.

Although there are some areas of land in private ownership yet to be reclaimed, that may be undertaken safely by private capital, the largest area is that of the public domain. Therefore, any effort to reclaim these lands must in some way be connected with the Federal Government. It, therefore, remains to point the way whereby this may be successfully done, and these great waste places be made a source of wealth to the people of this Nation and help supply their need as the time may come therefor. This question has engaged the attention of Congress for many years and numerous measures have been introduced, but as yet no one measure has met with general approval.

In view of all this, it may appear presumptuous to offer a suggestion, but the speaker will take the risk. The Federal Reserve Banks, the Federal Land Banks, and the Joint Stock Land Banks are all examples of business enterprises fostered by the Federal Government, and they have functioned well. Can a way not be found in them to meet the reclamation problem? In them a certain public service is performed under private direction and by private capital, supported as necessary by the Federal Government.

Why should Congress not create a corporation, the capital stock, at least in part, to be contributed by the Government, and its general powers, rights, and duties to be fixed by the Act? Its board of directors or managers, or a part of them, could be appointed by the President. Such corporation should be independent of any bureau, but connected with the Department of the Interior or Department of Agriculture in much the same way as the Reserve Banks are connected with the Treasury. The business of such corporation should be the reclamation of the waste lands of the country, either those of the public domain or State, or in private ownership, and whether by irrigation, drainage, or other means, the work should be so ordered and carried on from time to time as the demands on the agricultural resources may require, and not to the prejudice of the present farmed areas, by unduly increasing the lands available for cultivation, and under such terms and conditions as to assure a reasonable return on the undertaking. The lands should be disposed of in such a manner as to pass directly to actual farmers, on such terms as can be easily met by the proper use of the land. It would necessarily follow that no work should be undertaken where these conditions could not be reasonably met. Such corporation should have the power to issue its own bonds which should enjoy the same privileges and relation to the Federal Government as the bonds of Federal Land Banks, and should take over, complete, and liquidate the existing Government projects.

The advantages of such a corporation would be to take the reclamation problem out of politics. It would assure the orderly working out of these problems as the growth of the Nation demanded, and not to the harm of the agricultural population. It would secure the advantages of private skill and enterprise. It would lead to a continuing policy, improved and developed with

time and experience, regardless of changing administrations, yet it would enjoy Government support and restraint in case of need. It would work great economy over the present method. It would continue to function as long as the opportunity existed to add to, or improve, the areas capable of agricultural production, and would carry on the work when and as the needs demanded.

The foregoing suggestion is presented, without attempting to go into details, in the hope that it may set in motion some satisfactory solution of this problem. The conclusion is, therefore, that for the present private capital can only venture in assisting those projects in private ownership which come under the general rules previously stated; also in the refinancing of proved going projects, or in financing their improvement and further development. Should Congress see fit to provide for a corporation such as that suggested, or something better, a way may be opened for the further safe utilization of private capital.

Let us profit by past experience; it has been an expensive teacher, but out of it has come the knowledge of the weakness of the methods employed to put the desert to profitable use. Just as surely as the day will come when these lands will be put to beneficial use will engineers be able to outline a safe and sane, orderly, and timely way to do it. Until this is done, except to the limited extent noted, irrigationists are not in a position to invite private capital to invest in this great work.

PRESENT POLICY OF THE UNITED STATES BUREAU OF RECLAMATION REGARDING LAND SETTLEMENT*

By Elwood Mead,† M. Am. Soc. C. E.

Twenty-three years ago, Congress passed an act for the reclamation of arid lands by irrigation. Since that time, about $200 000 000 has been spent in the construction, operation, and maintenance of canals and reservoirs, and the building of drains. This money has come principally from the proceeds of sales of public lands, leases of oil land, and repayment of construction, operation, and maintenance costs by settlers. For all practical purposes it has added another State to the Union. Homes have been made available for 130 000 people. The farms created out of land once worthless are valued at $300 000 000 at least, and the towns and cities have had a great increase in taxable wealth. The crops produced in 1924 are valued at $66 500 000 and this annual value will be increased when advancing technique in agriculture and further subdivision of land have persuaded the reformed cowboy that irrigation on horseback is out of date.

The first conception of the Act made reclamation mainly a matter of engineering and construction. It was the popular opinion that if water were provided, irrigated agriculture would follow, that settlement of land and farm development under canals could be left to take care of themselves. No funds were provided to aid in such settlement and the help which could be incidentally extended was limited.

In the line of its main activity, the Reclamation Service has made a record which entitles it to the grateful appreciation of the people of the United States. The loftiest dams in the world are on these projects. Many had to be built in remote localities and numerous difficulties had to be overcome in handling torrential floods and deep foundations. Speaking generally, the engineering works on the twenty-four projects of the Bureau form an enduring testimonal to the zeal and capacity of the engineers of the Reclamation Bureau. The writer does not believe there is any service in any country which has a more competent staff of designers than that at work in the Engineering Headquarters of the Reclamation Service in Denver, Colo.

The operation of these projects has been difficult because water has had to be delivered to beginners in irrigation, to whom its methods and practices were new. They had to overcome great obstacles in the preparation of their land, and do this in too many instances without adequate capital and without credit

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suited to their needs. To these harassed individuals, the project superintendent has been a conspicuous and patient target for bricks hurled when they had to hit some one. Few bouquets are bestowed.

As far as the plan in the construction of works is involved, the organization of the Reclamation Bureau has taken a permanent and satisfactory form. These features of the Bureau's operations have passed beyond the sphere of evolution and need not be further considered, but social and economic forces render it necessary that present ideas as to what the term, reclamation, should include, should be revised. The increase in construction costs growing out of the World War, the greater expense of everything that goes into the development and operation of a farm, and the depression in agriculture, have made the question of farm development and the productive value of water far more important than was the case when the original Act was passed. Together they render it inevitable that if future reclamation is to be made a solvent enterprise, plans for new projects must include settlement and farm development and they must be made an important part of the Reclamation program.

On five projects authorized by the last (68th) Congress, but on which construction has not yet begun, and on two on which works have just been completed to supply water to new areas, the estimated and known construction costs vary from $100 to $160 per acre, with an average of more than $125 per acre. The land that is to pay these costs is unimproved, unpeopled, unleveled sagebrush, most of it remote from markets, and in some instances having such broken surfaces that preparation for irrigation will be expensive. The investigations preliminary to the undertaking of these projects, included the determination, by competent boards familiar with local conditions, of the probable cost of changing this raw land into habitable farms, and on these seven projects these boards fix the minimum expenditure necessary at from $4 000 to $10 000 per farm, or, roughly, $100 per acre. Combining the cost of the water right and the development cost of the farm, the settler will face an expenditure or indebtedness of from $200 to $250 an acre, before he has grown a crop or has established the productive value of land and water in irrigation.

The question which has to be considered is: How will settlers regard such an outlook? Under the Reclamation Act as it now is, which makes no provision for the organization of these settlers, for giving them expert advice and direction in farm development, and provides no means of advancing money to supplement their own capital in farm development, will they consider it an opportunity? If left to carry out this hard and costly work where they have to shift for themselves, will they be attracted to these projects, and if they come, will they succeed? The experience on the older projects is overwhelmingly in the negative.

On one of the projects which has been in operation for fifteen years, where the soil is good, the water supply ample, and where skilled cultivators are growing high-priced crops, half the land to-day is unoccupied, 20% is cultivated by renters, and only 30% by owners, and they grow crops of low acreage value. The money that has been lost by the Government on this project in operating an undeveloped property, would have financed the development of the farms.

In one State where more than \$15 000 000 has been spent in the construction of works, the total income since the outset—and that on some projects embraces a period of fifteen years—has not paid operating expenses. On the contrary, if repayment for construction is ignored, there is a deficit of \$4 000 000. The reasons are delayed farm development and lack of an agricultural program. These examples might be multiplied. They furnish a weighty reason for determining in advance how the land is to be settled and how the farms may be made productive before the project is approved and construction begins.

There is another important reason for doing this. Two-thirds of all the land on the older projects was in private ownership when they were authorized. Inflation of land prices, misstatement of conditions and improper use of the arts of modern publicity led to these lands being unloaded on uninformed, over-sanguine people, many of whom had no farming experience and who broke down when confronted with the obstacles which they had to surmount.

The greater part of the land in four of these new projects is privately owned. In its dry state it has little value. It is doubtful whether the income from it more than pays the taxes. The owners are widely scattered. Nearly all lack money to improve and cultivate, and a still larger number lack any desire or intention to do so. What they seek is to have the Government close its eyes to the future, trust to blind chance, and spend millions of dollars in building reclamation works. If this is done, however, there is danger that it will be followed by an inflation of land prices similar to that which occurred on the old projects.

The relation of private ownership of land to the social and economic value of these works built with Government money cannot, therefore, be ignored, and there has recently been conducted an economic investigation in which many of these land owners have been interviewed to ascertain what they believe their land is worth, what they will sell for if the irrigation works are built, and what terms they will give the purchasers. They have also been asked to indicate, in case they do not sell, their ability and willingness to farm, or to turn their land over to the Government and form a part of a co-ordinated scheme of colonization.

The responses confirm past experience that unless land prices are fixed in advance and a program of settlement decided to its final result, the building of these works will involve the Government in heavy losses. Farm development will be so slow that not only will the Government be kept out of the return of the money invested for many years, but operation and maintenance expense will either be a ruinous burden on the settlers, or have to be paid out of the public treasury.

It would seem, therefore, that one of two courses should be adopted. Either the building of irrigation works should cease, or aid and direction in settlement and farm development should be made a part of the Reclamation program. If this latter is done along lines that are in any sense expedient, no doubt is felt that nearly all these projects will be made more quickly productive

than in the past, and that they will prove solvent enterprises. If this is done, the program must include the following:

1.—The farm must be small and must be intensively cultivated. Only in this way can returns be secured that will justify the large acreage investment.

2.—The people who cultivate these farms must be selected. They must have industry and thrift. Only the superman could succeed if he had to bear the whole burden of this indebtedness. He could not do it at all unless money were advanced from some source not now available, and a credit scheme applied to settlers without capital would be financially unsound. The possession of some money is in most instances an indication of the earning and saving ability required in this work.

Any development scheme will break down that is based on the idea that settlers can be secured who have sufficient capital of their own. The records of the Reclamation Bureau show that 70% of the applicants for farms have less than \$1 500 capital, and only about 10% have more than \$2 500. The number of settlers who have from \$5 000 to \$10 000 is so small that it must be ignored. If the settler with \$2 000 capital is accepted, who takes a farm that will require an expenditure of \$4 000 to make it a going concern, the additional \$2 000 that is needed must be provided from some source on terms entirely different from those which local banks or private enterprises can and do give. Money for changing the raw land into farms must be advanced on long-time payments at a low rate of interest.

If this is done, there must be on these projects, an agricultural or economic superintendent, who will oversee these advances and watch the expenditure of the money, because they will not be made as loans but as service, not turned over to be carried around in the pocket of a borrower, but used to pay for definite and necessary improvements. If such a plan as that is put into operation and the money is furnished by the Government, then the Government must own the land on which the improvements are made. In such a case, this advance for improvements is underwritten (1) by the capital provided by the settler, and (2) by the fact that it enhances the value of the land which is owned by the Government.

With such advances, to be repaid under an amortized plan, with an interest rate of, say, 4%, and with from twenty to forty years' time for repayment of the principal, the man with from \$2 000 to \$3 000 can see his way through. If he loves rural life and wants to become a member of a prosperous community, these projects will attract him. If this aid and direction is given in order to provide an opportunity for a home-seeker who is an actual settler, then the condition of cultivation by the owner should be maintained. It would be little less than a farce to furnish all this assistance with the idea that it was to enable a desirable family to be planted permanently on the land, and then permit the farm to be sold to a non-resident land owner, who would install a tenant.

The last (68th) Congress passed a bill providing for the selection of settlers and empowering the Secretary of the Interior to fix the amount of capital they must have and their other qualifications. The Irrigation Com-

mittees of both Houses held many hearings on the question of aid and direction which should be included in a settlement program. The result was a bill introduced in the Senate, by Senator Kendrick, and in the House, by Congressman Winter, which provided for the employment of competent persons to act as advisers to settlers in farm development, for the subdivision of lands into farms and farm workers' homes, and for making advances to settlers to enable them to complete their farm improvements up to $3 000, the settler to furnish 40% of the money for all improvements on which advances were made.

This bill passed the Senate and was favorably reported in the House, but encountered opposition from members of the House who believed that the State should take charge of the subdivision and settlement of the land and furnish the money needed for advances in development. Provisions requiring this were incorporated in appropriations for three new projects, and the Secretary of the Interior was authorized to incorporate such provisions in a fourth. This legislation was opposed, in part, because it applied only to a few projects and not to others, and by those who do not believe the State has the resources to enable it to carry out this part of the development. It will unquestionably have further consideration in Congress.

Whatever authority takes charge, no one who is familiar with conditions on the older projects and who realizes how much greater the expenses will be in the future, believes that a success can be made of reclamation under the methods of the past. Not only must the settler have financial aid, but there must be introduced into the social and economic life of these new communities things that will enable people, by working together, to make farming more profitable, to cut down expenses, and to give them a sense of permanence and security that is too often absent. Long-time payments, community and co-operative organizations, tend to lessen the migratory and speculative spirit that has played too large a part in the development of new farming communities.

LAND SETTLEMENT OF IRRIGATION PROJECTS*

By Augustus Griffin,† M. Am. Soc. C. E.

There are many different aspects and phases to the problems of land settlement on irrigation projects. The subject is treated with the intention rather to suggest ideas for consideration and discussion than to attempt any thorough treatment, even of one phase of land settlement. The word "promoter" has been used in a broad sense to include a Government, district, railroad, or private colonization agency. It is realized that the manner of financing, sale of land and water rights, collection of charges, and handling of settlers, varies so widely that in a brief paper suitable distinctions and qualifications of statements cannot be made to conform to all conditions.

There are probably very few irrigation projects under which closer settlement is not possible and desirable, but many of the settlement problems have in these cases been solved or have become modified by local conditions. This paper, therefore, will be directed more particularly to the large isolated projects, developed on desert or sparsely settled range or dry-farmed areas, where the existing population is a small factor in necessary settlement and where the settler is a pioneer.

As soon as the first dollar is spent on a project, the annual charges begin. First, there is interest, then maintenance, and the annual charges of management and operation of the system. A large part of, and possibly all, the original capital investment must be made before any returns can be had and the investment has been increased by accruals of interest and annual costs.

Only when water is actually available can settlement actually start. Settlers are a necessity, for the returns on the invested capital must be produced from the land through the medium of farmer settlers. Money, energy, and art are required to make a "new settler" out of an ordinary man, but the art has been well developed and when the money is available the energy is easily commanded and by well-tried methods settlers are obtained whenever economic conditions are favorable. The foundation is laid for one of the the first and most important settlement problems, namely, "dissatisfaction". A family which possibly had no intention of moving has been persuaded to sell, move a thousand miles or so to a new district, of which it knows nothing, to engage in business wholly new to it, under new surroundings, and among strangers. Possibly the finances were inadequate or the personalities and training of the members of this family such that they were foredoomed to failure or to a long struggle

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against difficulties. Yet the project has to be settled in order to pay the annual costs and return the investment (if possible), in addition to which such settlement is necessary or desirable as a step in National development, to provide traffic for a railroad, or to sell somebody's surplus land. The promoter then may have two definite objects in view: First, to secure a return of the investment; and, second, to populate and develop the project as a National enterprise or to create traffic or other kind of business. The pioneer settler often feels that the secondary objects, which do not directly benefit him, are so important that he, as an agent in accomplishing them, is entitled to special consideration.

If the settlers on a project are prosperous, have their farms paid for, or can see that they can pay in a reasonable time; in other words, if they are making money, there is no serious settlement problem. The problem, in fact, is to bring about this highly desirable state of affairs.

It is necessary, therefore, first to have settlers, and the proper kind to get and the proper way to get them are large subjects in themselves which will not be discussed in this paper. It is sufficient to repeat that the art of getting settlers is well developed. Most new settlers find themselves on the project that some colonization organization has chosen for them without any very clear consciousness of how they got there. Fortunately, many of them feel that they arrived there by their own choice and initiative and this undoubtedly reduces the percentage of dissatisfied ones. Within certain limits the newer the project and the less development it has, the easier it is to get settlers, as there is generally the talking point of large areas of cheap land to choose from, with chances for speculative increases in land values, and the absence of actual trials and failures permits a freer scope in painting the possibilities.

There is undoubtedly much room for greater care and judgment in selecting settlers, but under the most favorable conditions there will always be misfits and others who, for various reasons, are foredoomed to dissatisfaction or failure. It has become a saying that the first two crops of settlers will fail and that the third and following crops may achieve success on the pioneer work of the first two. This is not strictly true, and it is probably more nearly correct to say that the successes of the first two crops, even if comparatively few, do the pioneer work which makes a greater percentage of successes among the later comers.

As the first settlers come to a project, they face a complex and critical set of conditions, although more often than not they do not realize this. They are full of hope and confidence, based mostly on the rosy tales of salesmen. They are strangers in a strange land, among neighbors as new as themselves. Even if they have farmed before they are facing a new type of agriculture, and this is usually true even if they have been irrigation farmers before. Previous irrigation experience may not be much of an asset if it has been with a different irrigation practice, under different conditions. The writer has come to the opinion that the man who will succeed is the man with tenacity of purpose, adaptability, good common sense, and with the ability and inclination to work hard. It is likewise essential that he and his family be of one mind, for many failures are traceable to dissatisfaction or homesickness of the wife

or other members of the family. The previous occupation or finances are of lesser importance, although other things being equal, there is no question that a man with previous farming experience is a better prospect. A certain minimum amount of capital is necessary for any particular set of conditions and larger capital will undoubtedly enable a settler to get on a paying basis sooner if it is judiciously used. Too often, however, the settler with ample capital dissipates it and sets himself back by the amount of time consumed in so doing.

Conditions vary greatly on different projects. On some, the land is native sod which has to be broken; on others, it is desert land, possibly covered with brush. In others, the land lies so nearly perfect for irrigation that water may be spread satisfactorily without preparation; in others, an expenditure of capital, of time, labor, and money must be made in smoothing, leveling, grading, or checking the land before a crop can be planted. Expenditures equivalent to a year or more use of the land, and up to $50 per acre are not uncommon. If the settler purchases 40 to 160 acres of land it may easily be a number of years before he can have all of it prepared for irrigation. The proper sequence and proportions of the various preliminary operations and the economical use of time and money are very important and this is where many failures start.

On large projects it will take a number of years to complete settlement and it is desirable to settle it by units, preferably on the best or most favorably located land, giving, also, due consideration to operating economics. Scattered settlement increases annual costs and hampers the development of the social structure.

The crops which can be raised successfully and marketed profitably are among the most important considerations. Alfalfa is the one staple common to all projects in the United States and Canada, which can be grown successfully and for which markets can be most easily developed, but even alfalfa growing usually goes through a period of depression when the supply exceeds the local demands and before the livestock industry has caught up with it. Generally speaking, irrigated lands are adapted to a wide variety of crops which become a drug on the market after the local demand is satisfied and until the community is sufficiently organized to develop markets, or until the supplies are large enough to induce outside buyers or industries to come to them.

The new settlers must get acquainted with their neighbors, adapt themselves to the new country, learn its climate, amalgamate themselves into communities, create a social structure of schools, churches, business centers, etc., study agricultural practices, and learn to prepare land for irrigation and to practice the art of irrigation. In many cases they must evolve the practices of land preparation and irrigation adapted to their conditions of soil, topography, climate, crops, markets, etc. These usually tend to uniformity on one project, due to uniformity of conditions, patterning after practices on similar projects, and to the necessity of adapting them to the design of the irrigation system. For example, they will differ very materially on a system designed to deliver irrigating heads of 1 or 2 sec-ft. continuously from a system designed for heads of 15 sec-ft., or more, in rotation. The evolution may extend

through a long period, gradually changing from a continuous-flow system to a rotation system.

There is a natural and usually unavoidable tendency to excessive use and misuse of water in the early years, for which poor land preparation due to lack of knowledge, time, and capital, inexperience in irrigation, excessive water supply, and ignorance of the dangers of over-irrigation are contributing factors.

The next step in irrigation development then following is the appearance of seepage, a rising water-table, water-logged land, and alkali. These frequently appear on land that has just previously produced exceptionally good crops with little or no irrigation and may cause widespread consternation. Probably, in most cases, these conditions are not actually caused by avoidable waste of water, but the date of their appearance is advanced by it. Most distressing conditions may result. Farmers whose equity in the land is small may move away, others sell at a sacrifice, while others must stay and face conditions. Remedial measures are necessary, but there may be serious obstacles in the way. The price of land was probably based on the construction of the irrigation system only, no allowance being made for drainage and reclamation of injured land. The settlers place the onus on the promoters; the promoters disclaim it. If the land is largely paid for by the settlers, the promoters can wash their hands of responsibility. It may develop as a problem not directly associated with the irrigation system. A loss of time almost surely results during which the ability of much land to produce crops is impaired or destroyed. Eventually, it may require an organization of a district with the raising of additional capital for drainage purposes. Extensive studies are required and at the best there are many unsatisfactory and obscure features and many questions still unsettled with respect to drainage and reclamation of alkali lands on a paying basis. On many existing projects the water-logging and alkali problems have been solved, some fortunate ones have never had them, whereas on others it is so serious as to impair the soundness of the project, or to result in a lower type of agriculture. There are cases in which serious errors of judgment were made by the promoters with respect to the suitability of land for irrigation or to the economic soundness of the development.

Given sufficient money and authority, engineers can build an irrigation system, and, also, to many engineers in the past the design and building of the system has seemed an end in itself. It has given them opportunities to express their personalities and theories in design and execution. They have not recognized, or have ignored, the fact that the diverting dam, the canals, and the structures are only a few among the means to an end, and that every dollar they spend, must be (or is expected to be) returned with interest by the settlers who come after. These payments must be produced from the soil, using the irrigation water as an instrument in production. They must not only pay interest on the construction cost and eventually extinguish the principal, but they must also make payments on the land if it is priced separately, pay the annual operation costs, taxes, and farming costs, but must themselves live, with such degree of comfort, and with such of the social and material conveniences and luxuries of life and with such margin of income over expen-

diture as will make them feel that they have bettered, or stand to better, the previous condition and opportunities of themselves and their families. It is unfortunately true that too many expect more than is reasonable in the pioneer days and those who win soonest and surest the comforts and luxuries of life are those who practice frugality and self-denial during this period and who build up a capital reserve of productive land, equipment, and bank account.

The irrigation system then should be designed as an instrument of service to the farmers who will come after, and as cheaply as possible so that the farmers, who must eventually pay for it, will be burdened as little as possible. The design and construction of an expensive or untried type of structure to gratify the designer's vanity may prove to be a costly error. No irrigation system is used to full capacity in its early years, and cheap or even makeshift construction may well be justified, leaving enlargement of the system and permanent construction of many structures to be carried out in connection with maintenance operations as the need arises. It is often possible to enlarge canals by promoting scour, possibly causing the coarser material to deposit where it can be used to advantage in strengthening banks. Much of the finer material may be automatically disposed of by being carried to the irrigated land. It requires more courage, self-restraint, and good judgment to build a cheap system than to build an expensive one, especially if ample funds are available.

An irrigation system should be designed and built for service and it should also be maintained and operated for service. The watchword of every operating organization should be "service" for twenty-four hours every day—personal service to every irrigator. System is nowhere needed more than in delivering water. Rules, policies, and records are necessary, but these should be drawn to serve the best interests of the irrigators, individually and collectively, and not to suit the convenience or swell the pride of the operating organization or its head. The system should be operated so as to contribute in the greatest degree possible to the productiveness of the land and the prosperity of its purchaser. Every member of the operating organization, from the head downward, should be "ordinary folk" with the farmers so that they can achieve a sympathetic understanding of their personalities, problems, and aspirations, and, at the same time, show the farmer that certain rules and restrictions are necessary to an orderly conduct of business and in the interest of the community and the individual. One of the troublesome features of irrigation farming is that each farmer must give up some of this individuality and freedom of action to satisfy the relations of irrigators to each other.

An organization, or organizations, either established by the promoters or by public agencies for experimenting with and introducing new crops, educating the new farmers, developing markets, and guiding the evolution of farming and irrigation practices, is very useful if not absolutely necessary. Such an organization should keep in close touch with farmers, not only to disseminate new ideas, but also to gather and distribute the cumulative experience of the farmers themselves. The operating staff can, with advantage, be organized to perform such functions, but if an independent organization is developed, it

should be closely associated with and its work correlated with that of the operating staff.

If land is sold on small payments down and small annual payments thereafter, the farmer has, for some years, a very small equity in the land and, therefore, but little incentive to stay, unless he is getting good returns. If land is cheap, settlers will incline to purchase too much. If times are bad, and the settler gets discouraged, he has no anchor to hold him and can abandon his land with but little loss. In such cases it is more than ever necessary to pick settlers carefully, to be sure they are the right type, that they have at least a minimum of capital, and to have an educational organization to help them and supervise them carefully.

Settlers will cost the colonization department anything from a few dollars each up to several hundred, or even more than $1 000 each, and as little money as possible should be wasted in unproductive expenditures for this purpose. It is probable that in the first years many of the settlers will not meet their contract obligations. Good judgment is required in dealing with them. It is a distinct loss in money, time, and prestige if a settler who has cost several hundred dollars abandons his land. Therefore, each settler should be dealt with individually on the merits of his personality and his situation. It may be a good investment to make concessions to him, or, on the other hand, it may be the best kind of business and a real kindness to him to close him out. Every effort should be made to have each farmer increase his equity in the land by payments, even of small amounts, on his contract and by productive improvements to the land. The building up of attractive home surroundings is a strong tie in holding a settler, but the construction of buildings out of proportion to his means, or the purchase of unnecessary or high priced equipment and live stock is a drain on his resources and is the cause of many failures.

There are times when the promoter feels that it is to the interest of the settler and himself to forego payments when due, in order that the settler may use the money for farm development. It is the writer's opinion that this is almost always unwise. It increases the farmer's debt, leads him into ill-advised expenditures, and tends to dull his conscience with respect to his contract. At any time that a settler's contract obligations are increased by accruals of unpaid interest, operation, and maintenance charges, or taxes above the original principal amount, his affairs and, therefore, the affairs of the promoter, are in an unhealthy condition. The seller of the land is protected only by the margin between total contract obligation and the net price which he can realize from the land if it returns to him, and, in this, account must be taken of the time and cost of abrogating an existing contract, the time and cost of securing a new settler, and the risk that the new settler will fail, even as his predecessor. It is, therefore, desirable to keep the sale to each settler in good standing and one means to this end is to exercise definite, but not irritating, pressure to reduce the settler's obligation at regular intervals. It is the writer's opinion that the agency for this purpose should be located on the project with full authority to deal with each individual farmer, under the limitations of any general rules or policies that may be laid down.

Where land is sold on contract and the settler has no title to the land, or where it is covered by a prior lien, it may be difficult for him to secure legitimate financing for his annual needs. This is a matter which may deserve careful attention. Generally speaking, if he has title to the land and the lien is in the form of a bonded debt he will have less trouble with financing. If and when each of a large proportion of the settlers acquires the major equity in his land, the settlement problems are well on the way toward solution. If and when land values increase, land seekers become plentiful, and loans exceeding the outstanding contract obligations against the average parcel of land are readily available, the problems are largely solved. When these conditions have been attained they will be evidence of the facts that the combination of soil, water supply, irrigation system, location, transportation, and other contributing factors have been favorable, that industrious, wide-awake, and intelligent farmers and business men are located on the project, that economic conditions have been favorable or adverse conditions have been overcome, that crops have been produced and marketed profitably, and that what was originally a group of strangers in a strange land have become a community of friends and neighbors, with schools, churches, and attractive homes. When such a stage has been reached, the problems of the pioneer, which are the problems of land settlement, will be largely settled.

IRRIGATION DEVELOPMENT THROUGH IRRIGATION DISTRICTS*

By E. Courtland Eaton,† and Frank Adams,‡ Members, Am. Soc. C. E.

From the earliest settlement, co-operation has been the dominant factor in irrigation development in Western America.§ In one form or another, it has been the means whereby at least 50% of the lands now irrigated in the seventeen Western States have been provided with an irrigation water supply. It has occupied the great middle ground in irrigation between individual and partnership effort on the one hand, and commercial and Government effort on the other. Furthermore, throughout irrigation history, co-operation has been the stabilizing influence in the creation of American irrigation institutions and policies, for through it men have learned that in irrigation, the interests of the community far transcend the interests of individuals.

The irrigation district is the most effective agency for co-operation in irrigation development. To the will to co-operate it adds prescribed forms and methods of government, the power of taxation, and the guiding hand of the State. By these means it has established itself as the most generally accepted agency for financing and operating the works of irrigation too large or too costly for successful individual effort.

Nature of Irrigation Districts

In all essential particulars the irrigation district is an agency of the State. It has a political status comparable to that of a town or a county, that is, legally established boundaries, an elected governing board, the power to tax and to create bonded debt, and the power to construct and operate works, save only that its authority is limited to matters relating to irrigation and to such coordinate affairs as drainage and hydro-electric power.

Present Status of Irrigation Districts

Irrigation districts are found in all the seventeen Western irrigation States except Kansas and Oklahoma—more than 500 in all, of which 244 were operating, 37 under construction, and 159 in preliminary stages, according to a Government tabulation, in 1921. The total area embraced in irrigation districts in

Note.—Written discussion on this paper will be closed with the August, 1926, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.

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§ For a history of irrigation districts in California, 1887-1915, see *Bulletin 2*, California State Dept. of Eng. For an account of irrigation districts in all the Western States, see *Department Bulletin 1177*, U. S. Dept. of Agriculture, published in 1923.

that year, omitting those inactive, exceeded 11 000 000 acres, and the irrigation district bonds outstanding on December 31, 1921, approximated $105 000 000. Since then irrigation-district activity has been very considerable in a number of the States, particularly in California and Idaho. In these two States alone 48 additional districts have been organized since 1921, embracing more than 1 000 000 acres and involving proposed expenditures in excess of $20 000 000. For ten or twelve years the United States Bureau of Reclamation has favored the irrigation district as the agency through which it shall deal with the water users on its irrigation projects, and many of the Government projects have been organized into irrigation districts. It is now the settled policy of the Bureau of Reclamation, as recently announced by the Secretary of the Interior, that all Government projects shall be so organized. Furthermore, outside the Government projects, practically no new irrigation development of great importance is now attempted on any other basis of organization, that is, the irrigation district in some form, whether of the type and name now most prevalent, the "conservation" district of some States, the larger water storage districts recently organized in San Joaquin Valley, California, the water improvement districts of Texas, or the irrigation or conservancy districts formed under certain special legislative enactments.

State Control of Organization and Financing

When the first irrigation district law was passed—the Wright Act, enacted by California in 1887—the thought uppermost in mind was to give communities the power to carry forward irrigation development against the opposition of strong minority land owners. Apparently little thought was given to the possibility of unwise action by proponents of irrigation enterprises. Consequently, no adequate safeguards were included in the legislation. The experience of the first ten years in California established the need for a more conservative procedure and this was provided in 1897. Further experience in California, as well as in several of the other Western States, confirmed this need and also the need for State regulation not only of irrigation district organization, but also of financing and construction of irrigation district works. Such State supervision is now general. To cite California alone, 90% of the time of the State Engineer, all the time of one irrigation engineer, and occasional part time of several others, is consumed in investigation and supervision of irrigation and water storage districts. The results of this State supervision have been gratifying in all the States providing it. Although the State authorities charged with such supervision have the difficult task of safeguarding the interests of the communities, the States, and the investment market for irrigation district securities, they have succeeded so far as to restore irrigation district bonds as accepted investments not only in Western investment centers, but also in the larger markets of Chicago, Ill., New York, N. Y., and several other cities entirely outside the irrigation field. To be sure, these markets are conservative, but they should remain so in order that only sound issues shall find a sale and that the influence of a discriminating investment market shall be joined with that of supervising public officials in holding irrigation district

development to enterprises that are physically feasible and economically justified.

Typical Irrigation Districts

To those members of the Society who have not had contact with irrigation, it might be profitable at this time to describe very briefly a few typical irrigation districts. As the irrigation district activities of the writers have recently been in California, it is most convenient to cite in these brief descriptions several districts in that State. In California, with minor exceptions, irrigation districts have been formed to improve land already in production, or to re-finance enterprises initiated and carried to some considerable development by private capital, rather than, in the first instance, to reclaim desert areas. To that extent the California districts do not fully present all the conditions found in some of the Western States. California, however, by virtue of its varied climate in different areas, has widely different types of agriculture, and, consequently, a number of different types of irrigation systems. Fig. 1 shows the distribution of irrigation districts in the United States as of December 31, 1921.

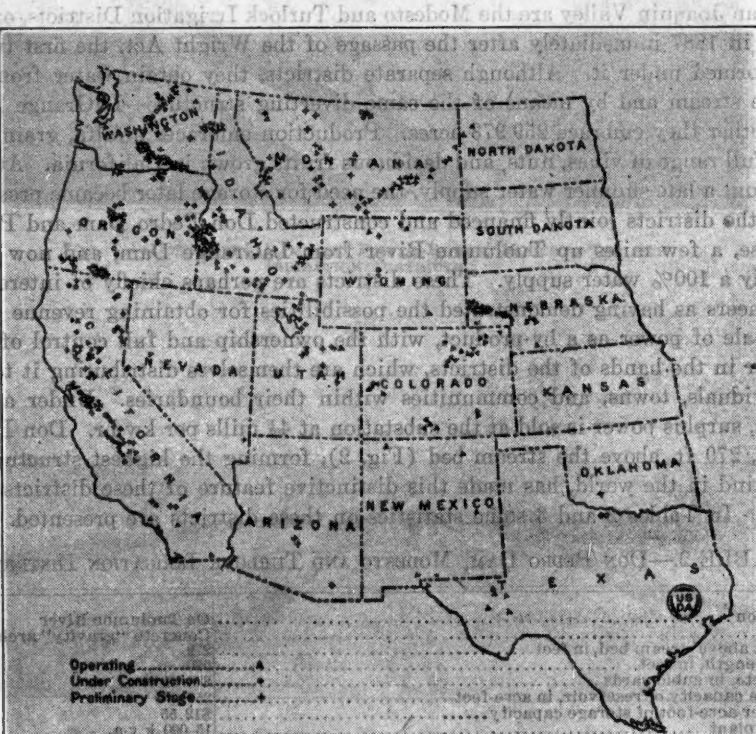


Fig. 1.—Distribution of Irrigation Districts in the United States, December 31, 1921.

Hot Springs Valley Irrigation District.—This district is in the extreme northeastern county of California at an elevation of about 4 500 ft., in the

region of the "horseback" farmer, using the crudest methods of irrigation. Here, water is applied to the wild meadow hay, or alfalfa fields, simply by flooding directly from Pit River or through inexpensive channels, diversion from the river being made by means of temporary earth, brush, or timber check dams thrown across the stream each year. The district organization was formed to finance the construction of an earth dam in Big Sage Valley to impound water for supplemental summer use. Table 1 gives some statistics on the Hot Springs Valley District.

TABLE 1.—HOT SPRINGS VALLEY IRRIGATION DISTRICT.

Area, in acres	9 640
Bonded debt per acre	$15.85
Water supply	Pit River and tributaries
Type of system	Gravity
Storage capacity of reservoir, in acre-feet	77 000
Principal crops	Meadow hay, alfalfa, grain
Duty of water, in acre-feet per acre	2 net
Average cost of water per acre per year	$1.53

Turlock and Modesto Irrigation Districts.—In the central part of the State in San Joaquin Valley are the Modesto and Turlock Irrigation Districts, organized in 1887 immediately after the passage of the Wright Act, the first two to be formed under it. Although separate districts, they obtain water from the same stream and by means of the same diverting structure—LaGrange Dam. Together they embrace 259 973 acres. Production embraces alfalfa, grain, and the full range of vines, nuts, and deciduous fruits grown in California. At first without a late summer water supply, the need for storage later became pressing, and the districts jointly financed and constructed Don Pedro Dam and Power House, a few miles up Tuolumne River from LaGrange Dam, and now have nearly a 100% water supply. These districts are perhaps chiefly of interest to engineers as having demonstrated the possibilities for obtaining revenue from the sale of power as a by-product, with the ownership and full control of this power in the hands of the districts, which are themselves distributing it to the individuals, towns, and communities within their boundaries. Under agreement, surplus power is sold at the substation at 4½ mills per kw-hr. Don Pedro Dam, 279 ft. above the stream bed (Fig. 2), forming the highest structure of the kind in the world, has made this distinctive feature of these districts possible. In Tables 2 and 3 some statistics on these districts are presented.

TABLE 2.—DON PEDRO DAM, MODESTO AND TURLOCK IRRIGATION DISTRICTS

Location	On Tuolumne River
Type	Concrete "gravity" arch
Height above stream bed, in feet	279
Crest length, in feet	980
Concrete, in cubic yards	278 810
Storage capacity of reservoir, in acre-feet	250 000
Cost per acre-foot of storage capacity	$12.55
Power plant	15 000 k.v-a.
Cost of power plant per kilovolt-ampere	$61.00

Lindsay-Strathmore Irrigation District.—Still farther south, in San Joaquin Valley, is a small but highly developed irrigation district—the Lindsay-Strathmore—representative of high annual costs and typifying the length to

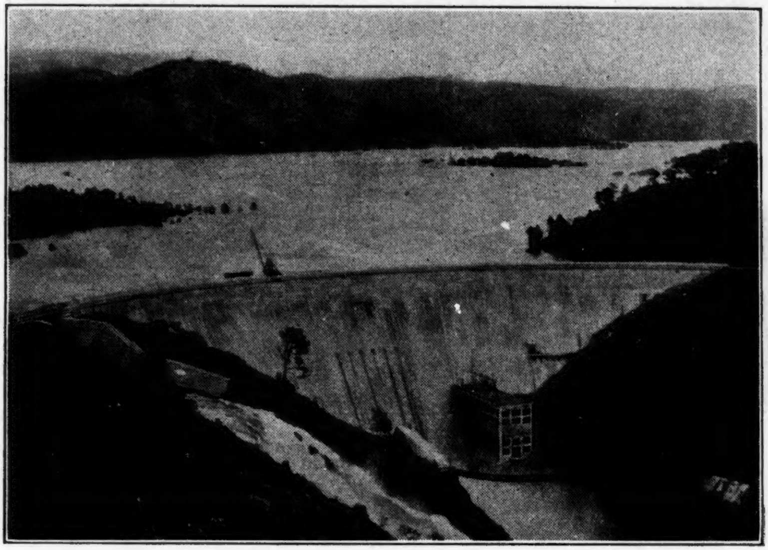


FIG. 2.—DON PEDRO DAM AND POWER-HOUSE, MODESTO-TURLOCK IRRIGATION DISTRICTS.

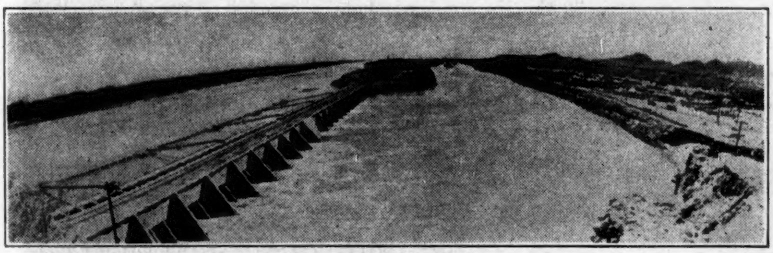
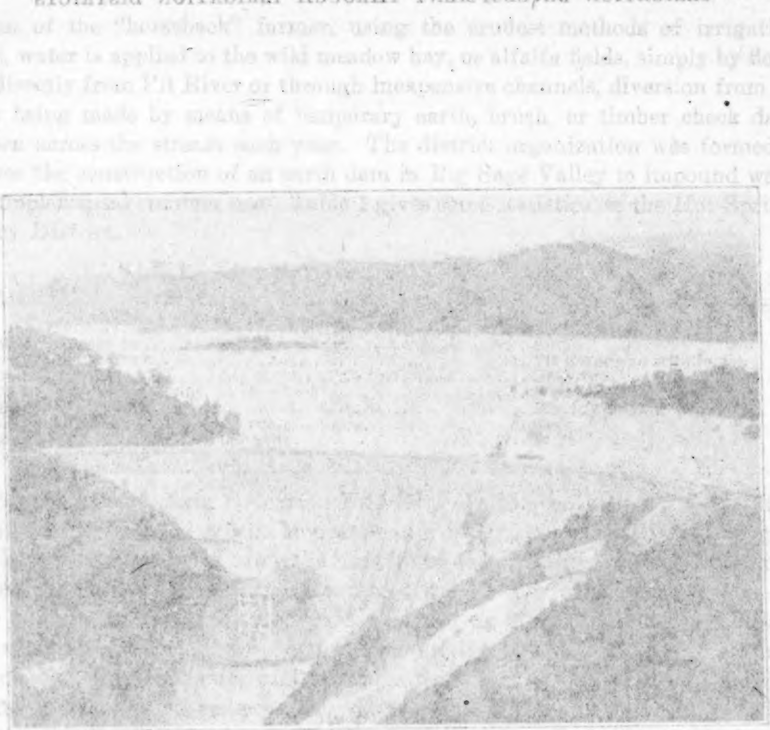


FIG. 3.—ROCKWOOD DIVERSION DAM, COLORADO RIVER, IMPERIAL IRRIGATION DISTRICT.

region of the "horseback" farmer, using the crudest methods of irrigation. Here, water is applied to the wild meadow hay, or alfalfa fields, simply by flooding directly from Pit River or through inexpensive channels, diversion from the river being made by means of temporary earth, brush, or timber check dams thrown across the stream each year. The district organization was formed to finance the construction of an earth dam in Big Sage Valley to impound water for supplemental summer use. Table 1 gives data relative to the Hot Springs Valley District.



Lindsay-Strathmore Irrigation District.—Still farther south, in San Joaquin Valley, is a small but highly developed irrigation district—the Lindsay-Strathmore—representative of high annual costs and typifying the length to

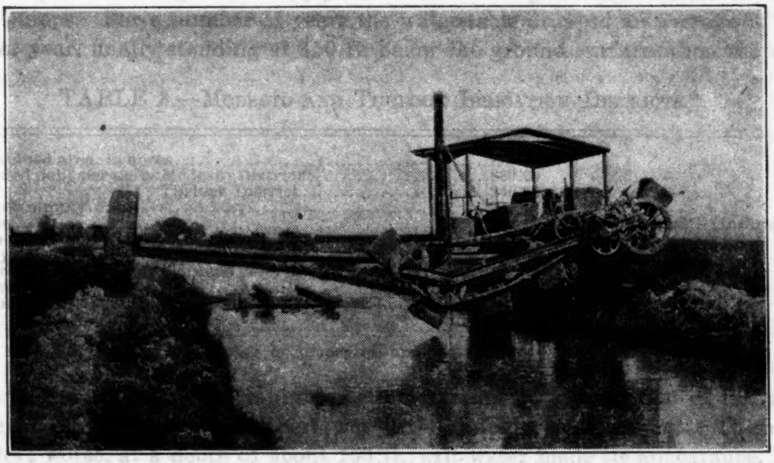


FIG. 4.—RUTH DREDGER REMOVING SILT FROM CANAL, IMPERIAL IRRIGATION DISTRICT.

FIG. 5.—VIEW OF IRRIGATED LAND, IMPERIAL IRRIGATION DISTRICT.

which the successful fruit grower feels he can go to obtain an adequate water supply. The development is an old one, principally citrus, commencing by irrigation from private wells. At first, centrifugal pumps set in shallow pits were effective, the lowering water supply later necessitating the substitution of deep well pumps. For a number of years the water-table dropped an average of 8.8 ft. per year, finally standing at 150 ft. below the ground surface when the district was organized. The rapid depletion was due to the location of the area close to the foothills where there was little annual replenishment. To make matters worse, at a depth of about 150 ft., salt water made its appearance, and in some cases as much as 22 lb. per tree per year was carried in the irrigation water, with disastrous results. Therefore, 1 100 acres of water-bearing land in the delta of the Kaweah River was purchased by the District and water was pumped from thirty-nine wells through concrete lined canals and a concrete flume (of special interest because built with a Gunite machine) and steel pipelines, from which it is metered to each consumer. The entire lift is 285 ft.

TABLE 3.—MODESTO AND TURLOCK IRRIGATION DISTRICTS.*

Combined area, in acres	259 973
Bonded debt per acre, Modesto District	$60.39
" " " Turlock District	$40.66
Water supply	Tuolumne River.
Type	Gravity distribution with storage.
Principal crops	Alfalfa, vines, deciduous fruits.
Duty of water, in acre-feet per acre	2.5, net
Average cost of water per acre per year:	
Modesto District	$5.86
Turlock District	$3.73

* Hydro-electric power is generated and distributed by these Districts.

TABLE 4.—LINDSAY-STRATHMORE IRRIGATION DISTRICT.

Area, in acres	15 285
Bonded debt per acre	$107.95
Type	Pumping from wells with pipe distribution
Principal crop	Citrus fruits
Duty of water, in acre-feet per acre	1.6, net
Average cost of water per acre per year	$26.24

Imperial Irrigation District.—Farthest south in California, next to the Mexican border, is the Imperial Irrigation District (Figs. 3, 4, and 5), in the land of the real desert, where absolutely no economic production would be possible without irrigation. This is the largest irrigation district in the United States. Here, rainfall is hated, because it is never sufficient to have value and is a nuisance in gumming up the roads. Irrigation is practiced all the year round. After the desert hummocks were removed and the land was leveled, grain was planted as the breaking crop, followed by alfalfa. In the World War period, cotton was the chief single crop and is still largely grown, but the Valley is in the main one of diversified production, with cantaloupes, lettuce, and asparagus for the early markets perhaps outstanding. The most interesting engineering features of this enterprise are associated with its control of the Colorado River and its unusual silt problems. The water, diverted from the Colorado,

contains 0.6% by volume of silt and sand which are removed from the canals by dredging and sluicing. The sluiced water is passed into Salton Sea, 250 ft. below sea level, as is the drainage water also. The process presents a duplication in miniature of the building up of the deltas at the mouth of the Colorado. The district maintains more than 60 dredges and 1 300 men in the annual clearing of its canals and laterals from Colorado River silt deposits, which, if not removed, raise the canal beds from 6 to 12 in. per year. Statistics on Imperial Irrigation District are given in Table 5.

TABLE 5.—IMPERIAL IRRIGATION DISTRICT.

Area, in acres	605 000
Bonded debt per acre	$26.45
Water supply	Colorado River
Type	Gravity distribution without storage
Principal crops	Alfalfa, cotton, cantaloupes, lettuce
Duty of water, in acre-feet per acre	3, net
Average cost of water per acre per year	$5.50
Annual cost of silt removal	$1 000 000

SOME CONCLUSIONS

Engineers are interested in irrigation districts because at present, and apparently in the future, those who are working in irrigation, whether on investigations, construction, maintenance, and operation, irrigation machinery and equipment, and to a very considerable and increasing extent, on hydroelectric development, will find themselves dealing with irrigation districts. Some of the obvious conclusions from irrigation district history, therefore, should be of interest.

First, the irrigation district cannot safely be used as the irrigation agency in uneconomic land developments or speculation. This has been tried over and over again and in every case failure has resulted. Although in some of the States the irrigation district form of organization must be resorted to in order to initiate projects involving the reclamation of arid lands, the proceeding is exceedingly difficult and hazardous. Until 1921, inclusive, 58% of all districts formed were for supplemental construction or the acquirement of existing systems.*

The ideal conditions for the organization of an irrigation district is to find the enterprise already under way, although not adequately financed; sufficient settlement and sufficient voting population to insure a *bona fide* decision on the organization and financing program on the basis of real merit as contrasted with speculation; and established agricultural possibilities. State regulation of irrigation district organization is an absolute requirement if unsound district ventures are to be stopped at their inception—projects with inadequate water supply, unproved agricultural possibilities, uneconomic costs, and unpromising opportunities for settlement. In deciding upon the feasibility of an irrigation district project agricultural and economic factors of production and settlement are equally important with good engineering.

* *Department Bulletin* 1177, U. S. Department of Agriculture, p. 54.

In planning an irrigation district system the only safe and fair course is to have the plan comprehend not only the immediate needs of the district, but also those that will be required for the completion of the system even if not to be constructed for some years to come. Experience has proved it to be far safer to tell settlers and irrigators the whole truth about what irrigation water will cost them when the system, including storage, distribution features, and necessary drainage, is finally installed.

Although the market for irrigation district bonds has been gratifyingly receptive to sound issues up to several millions of dollars, there is not yet a satisfactorily competitive market for the larger issues such as those recently put out by some of the districts. Lack of competition among bond dealers recently made one irrigation district take a discount of 4% on a $10 000 000 issue, and part of it brought a 6% profit to the dealers. This is more than irrigation can stand, and some way must be found to supplement the credit of districts proposing such large issues.

Although the ultimate authority in the management of irrigation districts is placed by law in the boards of directors, very properly subject in most States to a measure of State control of financing and construction, the real leadership must be furnished by the firm, intelligent, and tactful engineer.

It is the engineer who, through his skill, must hold the irrigation district in safe bounds and make it go; who must supplement the lack of business experience of the average board of directors by his judgment; who above all must keep his equanimity and carry on in spite of the all too frequent ill-support and lack of understanding by land owners. It is the engineer who must ever have in mind that however efficient his services, the professional relationship between the engineer and his irrigation district client is still a more or less unstable one, because the human beings who constitute irrigation districts, like the rest of us, have not yet learned all the lessons of co-operation.

TREND OF CONSTRUCTION COST OF CERTAIN PUBLIC UTILITIES

By William Breuer,* Jun. Am. Soc. C. E.

Synopsis

A valuation was recently made of one of the largest public utilities in the United States and in connection therewith a detailed study of the trend, year by year, from 1914 to date, of the construction cost of various public utility properties.

It was found that, since 1918, whether a street railway, a gas or an electric property, a power plant or a distribution system, a sub-station or a transmission line, for all of these, regardless of location or magnitude, the composite trend was substantially the same. In no instance was an exception found.

An appraisal of a large electric light and power property was made about a year or two ago, as of present-day prices; and for retirement purposes an investigation to determine approximately the original cost of any property in question was also made.

It is the practice in making such analyses to use either 1913 or 1914 price levels for a base. We are, however, in a new and normal period of post-war prices and because more accurate results are obtained by the use of present-day cost data, a 1924 base was used. Furthermore, 1913 was a year of depression, and in order to determine whether the results obtained were typical of utilities as a whole, other light, power, and street railway properties in various parts of the country were also trended.

The electric light and power property was, of course, first analyzed, and it was necessary to adhere to the Classification of Accounts prescribed by the Public Service Commission. A separate trend was compiled for each of more than thirty accounts, and a composite trend derived by combining into a weighted average these thirty accounts. The trends for these accounts were not at all uniform which was also true of each item of each account.

In a similar manner a gas property was analyzed, and as its composite trend was substantially the same as that of the electric light and power property, it was thought that this was merely a coincidence. A street railway property was then studied, but its composite trend did not differ greatly from that of the gas or electric property. Other light and power and street rail-

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* Engr., Day & Zimmermann, Inc., Philadelphia, Pa.

way utilities were also analyzed, but regardless of location or magnitude no exception was found.

A study was likewise made of the trend of steam power plants, the plants studied representing a wide range of conditions and many stages in the progress of the art of power plant design. Some were equipped with turbo-generators, others with engine-driven generators, with conversion equipment for street railway purposes, and even with ice plant machinery, and the installations ranged from capacities of 120 to more than 100 000 kw. A study was also made of electric distribution systems and of transmission lines, but again in no instance was an exception found, in that all power plant trends were about the same and all distribution trends the same.

Several months were necessary to make the analyses and at no time were references made to other sources of information. Of course, the research work was very exhaustive. The material trends used were those secured from manufacturers which were further checked by means of unit costs from purchase order records.

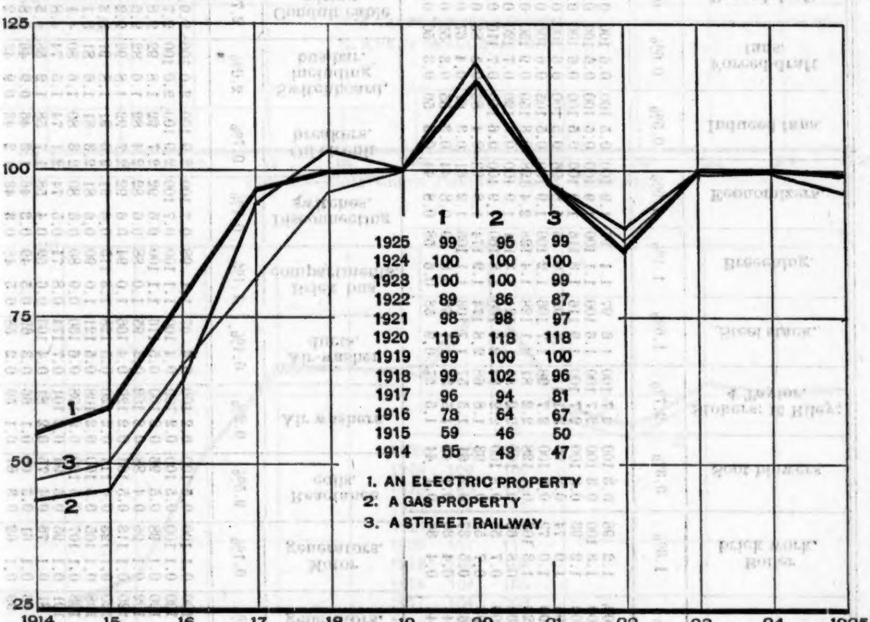

FIG. 1.—COMPOSITE PUBLIC UTILITY TRENDS.

So much of the data are also confidential and not for publication that it is rather difficult to include the fundamental facts from which the conclusions have been derived. There is, however, submitted a tabulation, Table 1, in which the method of procedure is shown in detail. Table 1 is typical of the analyses, and in order to show more clearly the results obtained, several charts have also been prepared.

On one of these, Fig. 1, composite trends typical of street railway, gas, and electric properties have been plotted, and the uniformity from 1919 to 1925,

TABLE 1.—COMPOSITE TREND FOR A STEAM POWER PLANT.

Year.	Power plant building.	Yard improvements.	Boilers, B. & W. No. 20.	Boiler brick work.	Soot blowers.	Stokers: 16 Riley; 4 Taylor.	Steel stack.	Breeching.	Economizers.	Induced fans.	Forced-draft fans.	Forced-draft air-ducts.	Feed-water heaters.	Pumps.	Tanks.	Piping, meters, gauges.	Coal-handling machinery.	Locomotive cranes.
	34.6%	3.1%	6.9%	1.3%	0.3%	2.7%	1.6%	1.1%	1.9%	0.5%	0.6%	0.3%	0.6%	0.3%	0.1%	8.0%	3.0%	0.4%
1925	33.9 98	3.0 98	6.9 100	1.2 96	0.3 100	2.7 100	1.6 97	1.1 97	1.9 100	0.5 100	0.6 100	0.3 97	0.6 100	0.3 100	0.1 97	8.0 100	3.2 105	0.4 96
1924	34.6 100	3.1 100	6.9 100	1.3 100	0.3 100	2.7 100	1.6 100	1.1 100	1.9 100	0.5 100	0.6 100	0.3 100	0.6 100	0.3 100	0.1 100	8.0 100	3.0 100	0.4 100
1923	34.6 100	3.1 100	6.9 100	1.2 93	0.3 100	2.7 100	1.9 116	1.3 116	1.9 100	0.5 100	0.6 100	0.3 116	0.6 96	0.3 99	0.1 116	8.0 100	2.9 95	0.4 93
1922	30.4 88	2.7 88	6.6 96	1.0 74	0.3 96	3.1 116	1.5 95	1.0 95	1.8 96	0.5 100	0.6 100	0.3 95	0.6 96	0.3 99	0.1 95	7.4 92	2.6 87	0.3 84
1921	32.5 94	2.9 94	6.9 100	1.0 79	0.3 100	2.7 98	1.7 106	1.2 106	1.9 100	0.5 105	0.6 105	0.3 106	0.6 95	0.4 124	0.1 106	8.1 101	3.3 111	0.4 104
1920	39.8 115	3.6 115	8.8 128	1.3 103	0.4 128	2.2 81	2.1 128	1.4 128	2.4 128	0.8 150	0.9 150	0.4 128	0.6 108	0.4 141	0.1 128	9.4 118	3.8 125	0.5 124
1919	31.1 90	2.8 90	6.9 100	0.8 68	0.3 100	2.2 81	1.9 121	1.3 121	1.9 100	0.6 120	0.7 120	0.4 121	0.6 105	0.4 128	0.1 121	8.6 107	3.3 111	0.4 111
1918	27.7 80	2.5 80	6.9 100	0.7 56	0.3 100	2.2 83	2.6 160	1.8 160	1.9 100	0.6 110	0.7 110	0.5 160	0.5 83	0.3 113	0.2 160	8.0 100	3.2 105	0.5 137
1917	24.6 71	2.2 71	6.4 93	0.7 53	0.3 93	1.8 68	2.8 174	1.9 174	1.8 93	0.4 83	0.5 83	0.5 174	0.4 66	0.2 81	0.2 174	7.2 90	3.0 100	0.5 123
1916	20.4 59	1.8 59	4.2 61	0.5 37	0.2 61	1.5 57	1.7 103	1.1 103	1.2 61	0.3 67	0.4 67	0.3 103	0.3 57	0.2 70	0.1 103	5.8 72	2.2 71	0.3 66
1915	17.0 49	1.5 49	3.0 44	0.4 30	0.1 44	1.5 57	1.1 69	0.8 69	0.8 44	0.3 55	0.3 55	0.2 69	0.3 57	0.2 63	0.1 69	4.4 55	2.1 69	0.2 51
1914	16.6 48	1.5 48	3.0 44	0.4 30	0.1 44	1.5 57	0.9 58	0.6 58	0.8 44	0.3 50	0.3 50	0.2 58	0.3 57	0.2 63	0.1 58	4.4 55	2.0 65	0.2 52

Year.	Locomotives.	Ash hoppers.	Condensers.	Turbo-generators.	Motor generators.	Reactance coils.	Air washers.	Air-washer ducts.	Brick bus compartments.	Disconnecting switches.	Oil circuit breakers.	Switchboard, including bus-bar.	Conduit cable wiring.	Power transformers.	Current transformers, also potential.	Insulators.	Lightning arresters.	Composite.	
	0.3%	0.5%	4.5%	14.3%	0.1%	0.5%	0.2%	0.4%	1.1%	0.7%	0.7%	2.0%	2.7%	0.3%	0.3%	0.7%	0.2%	99.0%	
1925	0.3	0.5 100	4.4 97	14.3 100	0.1 100	0.5 100	0.2 100	0.4 97	1.1 98	0.7 100	2.9 100	2.0 100	2.9 107	0.3 100	0.3 100	0.6 89	0.2 100	98.8 = 99	1925
1924	0.3 100	0.5 100	4.5 100	14.3 100	0.1 100	0.5 100	0.2 100	0.4 100	1.1 100	0.7 100	2.9 100	2.0 100	2.7 100	0.3 100	0.3 100	0.7 100	0.2 100	99.0 = 100	1924
1923	0.3 104	0.5 100	4.5 100	14.3 100	0.1 98	0.5 96	0.2 100	0.5 116	1.1 100	0.6 92	2.7 92	1.8 92	2.8 103	0.3 96	0.3 96	0.7 100	0.2 96	99.0 = 100	1923
1922	0.3 94	0.5 96	4.2 94	14.6 102	0.1 95	0.4 88	0.2 102	0.4 95	1.0 88	0.6 82	2.4 82	1.6 82	2.5 91	0.3 89	0.3 89	0.7 95	0.2 89	91.4 = 92	1922
1921	0.3 100	0.5 100	4.6 102	17.2 120	0.1 113	0.5 104	0.2 120	0.4 106	1.0 94	0.6 92	2.7 92	1.8 92	2.2 81	0.3 105	0.3 105	0.7 105	0.2 105	99.0 = 100	1921
1920	0.3 119	0.6 128	5.8 129	17.2 120	0.1 123	0.6 114	0.2 120	0.5 128	1.3 115	0.6 91	2.6 91	1.8 91	3.3 121	0.3 114	0.3 114	0.9 131	0.2 114	115.5 = 117	1920
1919	0.3 98	0.5 100	5.0 110	15.5 108	0.1 105	0.5 100	0.2 108	0.5 121	1.0 90	0.6 81	2.3 81	1.6 81	3.1 116	0.3 100	0.3 100	0.7 93	0.2 100	97.0 = 98	1919
1918	0.3 95	0.5 100	5.4 121	15.5 108	0.1 107	0.5 100	0.2 108	0.6 160	0.9 80	0.6 80	2.3 80	1.6 80	4.1 151	0.3 101	0.3 101	0.6 85	0.2 101	95.1 = 96	1918
1917	0.3 97	0.5 93	4.5 99	14.5 101	0.1 95	0.5 94	0.2 101	0.7 174	0.8 71	0.5 74	2.1 74	1.5 74	4.8 178	0.3 94	0.3 94	0.7 94	0.2 94	87.9 = 89	1917
1916	0.2 66	0.3 61	3.5 77	11.6 81	0.1 76	0.4 75	0.2 81	0.4 103	0.6 59	0.4 58	1.7 58	1.2 58	4.5 166	0.2 75	0.2 75	0.4 52	0.2 75	68.6 = 69	1916
1915	0.1 46	0.2 44	3.0 67	9.5 66	0.1 61	0.3 65	0.1 66	0.3 69	0.5 49	0.3 46	1.3 46	0.9 46	2.8 104	0.2 65	0.2 65	0.3 36	0.1 65	54.5 = 55	1915
1914	0.1 43	0.2 44	2.7 59	8.5 99	0.1 62	0.3 61	0.1 59	0.2 58	0.5 48	0.3 48	1.2 43	0.9 48	2.2 81	0.2 61	0.2 61	0.3 42	0.1 61	51.5 = 52	1914

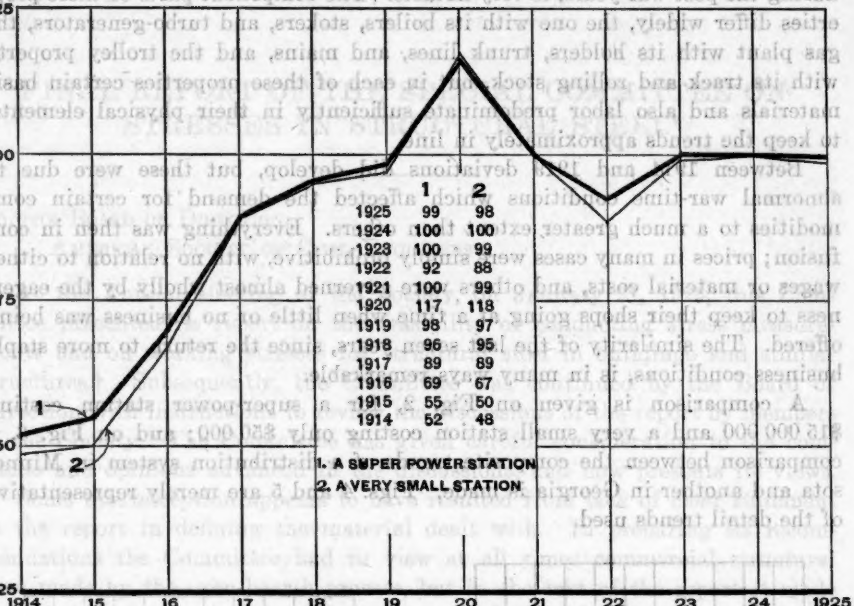

FIG. 2.—COMPOSITE POWER PLANT TRENDS.

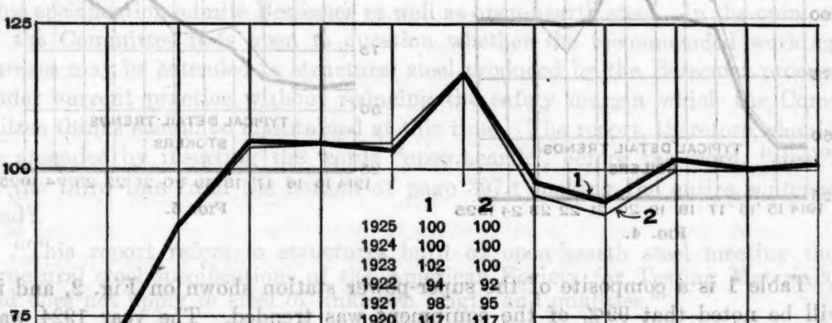

FIG. 3.—COMPOSITE DISTRIBUTION LINE TRENDS.

during the post-war years, is very notable. The component parts of these properties differ widely, the one with its boilers, stokers, and turbo-generators, the gas plant with its holders, trunk lines, and mains, and the trolley property with its track and rolling stock, but in each of these properties certain basic materials and also labor predominate sufficiently in their physical elements, to keep the trends approximately in line.

Between 1914 and 1919 deviations did develop, but these were due to abnormal war-time conditions which affected the demand for certain commodities to a much greater extent than others. Everything was then in confusion; prices in many cases were simply prohibitive, with no relation to either wages or material costs, and others were governed almost wholly by the eagerness to keep their shops going at a time when little or no business was being offered. The similarity of the last seven years, since the return to more staple business conditions, is in many ways remarkable.

A comparison is given on Fig. 2, for a super-power station costing $15 000 000 and a very small station costing only $50 000; and on Fig. 3, a comparison between the composite trends of a distribution system in Minnesota and another in Georgia is made. Figs. 4 and 5 are merely representative of the detail trends used.

FIG. 4.

FIG. 5.

Table 1 is a composite of the super-power station shown on Fig. 2, and it will be noted that 99% of the equipment was trended. The year 1924 was used as a base of 100, and multipliers were compiled for each of the items, year by year, from price trend letters. The composite summary at the end of Table 1 is not predetermined, but is the aggregate of the weighted averages of each of the thirty-five major items.

That all these utilities, power plants, and distribution systems would trend so much alike, was not expected, but if similar research work of others were readily available, the writer is very confident that their experience would be but a substantiation of these findings.

FINAL REPORT OF THE SPECIAL COMMITTEE ON STRESSES IN STRUCTURAL STEEL*

To the Board of Direction,

American Society of Civil Engineers:

At the Annual Meeting of the Society, on January 21, 1925, this Committee presented its report on the feasibility of conducting stress measurements and on working stresses for structural steel in buildings and similar structures.† Subsequently, the Committee was continued by the Board of Direction with instructions to review the discussions of the report by members of the Society. The Committee has given careful consideration to the comments and opinions advanced in the discussions, and now presents its views.

Some misconception appears to have resulted from lack of clear statement in the report in defining the material dealt with. In preparing its recommendations the Committee had in view at all times commercial structural steel made by the open-hearth process, but in the text of the report it made use of the specification for structural steel for buildings of the American Society for Testing Materials (A9-24) to define the material under discussion. This specification admits Bessemer as well as open-hearth steel. In the opinion of the Committee it is open to question whether the recommended working stresses may be extended to structural steel produced by the Bessemer process under current practice without reducing the safety margin which the Committee thinks should be maintained at this time. The report, therefore, should be amended by inserting the words "open-hearth", before the word, "steel", in the third line from the bottom of page 397,‡ making the entire sentence read:

"This report refers to structures built of open-hearth steel meeting the structural steel specifications of the American Society for Testing Materials, and does not apply to steel of unknown origin and qualities."

With respect to other phases of the discussion the Committee finds that all the essential points brought out had previously been discussed at considerable length within the Committee before it formulated its conclusions on working stresses. So far as these points appeared to be of practical significance, they were dealt with in the report, although necessarily in brief form. It is believed that the text of the report supplies an answer to the criticisms and questions contained in the discussion, and that further reply would be mere repetition.

There is one general view reflected in several of the discussions, however, which the Committee feels to be dangerously unsound and to which it there-

* Presented to the Annual Meeting, January 20, 1926.

† *Proceedings*, Am. Soc. C. E., March, 1925, Papers and Discussions, p. 392.

‡ *Loc. cit.*

fore recurs. It is urged by some members in discussing the report that the design of structural steel framework may not in all cases be in competent hands, and that for this reason working unit stresses should be held down to a somewhat lower limit than might otherwise seem permissible; or, to restate the matter, it is argued that a lowered working stress makes incompetence tolerable, and secures to the public as great a measure of safety as would be obtained through fully competent technical service using somewhat higher working stresses. The Committee feels strongly that abundant experience brands this view as fallacious and a source of great danger, at least in the field of structural steel. Whether in other fields incompetence may be neutralized by increased safety margins, the Committee is not prepared to state. In the field of steel construction, however, it is thoroughly established that the forms in which incompetence manifests itself bear no relation to unit stress limits; nor would any reasonably practicable limitation of working unit stresses afford protection against the results of such incompetence. For this reason, the course suggested by those engineers who would hold to relatively low working stresses because there may be an occasional incompetent designer, would not only prove ineffective but, in the long run, would tend to promote danger, through encouraging the belief that limitation of working stresses disposes of the danger from incompetence.

In its report submitted in 1925, the Committee, therefore, laid down the principle that all steel designing practices should be based on the assumption of reasonable competence. It desires to emphasize this statement in this report.

A high degree of skill in design will generally result in more efficient utilization of material than a more ordinary degree, and to this extent the highly skilled engineer may attain a given measure of safety with somewhat higher working stresses than his colleague of somewhat less skill. Such distinctions were not in the minds of the Committee, however, in its references to competent and incompetent design. The recommendations contained in its report were framed to be used not only by the designer of unusual skill, but by the competent designer of that degree of skill customarily found in practice. These working stresses are not safe for use by the incompetent designer, nor is any other known scale of working stresses safe under this condition.

The Committee respectfully asks to be discharged.

F. O. Dufour, *Chairman*,
R. J. Fogg, *Secretary*,
H. G. Balcom,
J. H. Edwards,
Robert Farnham,
L. D. Rights,
F. E. Schmitt,
W. J. Thomas.

November 12, 1925.

Minority Report

The undersigned members of the Committee did not concur in the recommendation for working stresses in structural steel based on 20 000 lb. per sq. in. unit stress in tension, contained in the report of the Committee submitted to the Annual Meeting on January 21, 1925. The reasons for dissenting from that part of the report were stated in the Minority Report.* They do not agree that all the points raised in the discussion were adequately covered in the previous report, nor that discussion of so important a matter should be dismissed so briefly. They believe that additional consideration should be given to this subject, and further knowledge of the basis of working stresses and further control of materials secured, before working stresses higher than those proposed in the Minority Report can properly be recommended for use in regular practice.

They concur in the request for the discharge of the Committee.

CLEMENT E. CHASE,
O. F. DALSTROM,
F. M. MASTERS,
L. J. TOWNE.

November 18, 1925.

* *Proceedings*, Am. Soc. C. E., March, 1924, Papers and Discussions, p. 404.

FINAL REPORT OF THE SPECIAL COMMITTEE ON IMPACT IN HIGHWAY BRIDGES*

To the American Society of Civil Engineers:

In submitting a final report the Committee does not presume to say that the present information on impact in highway bridges is such that nothing more remains to be accomplished. It is the belief of the Committee, however, that the available data have reached the point where they suggest definite conclusions of sufficient precision to serve as a reliable guide in design, at least as far as floors are concerned.

Introduction

The chief sources of information have been: First, a co-operative project on impact in highway bridges at Ames, Iowa, between the United States Bureau of Public Roads, the Iowa State Highway Commission, and the Engineering Experiment Station of Iowa State College;† second, extended determination of impact blows of truck wheels by the U. S. Bureau of Public Roads;‡ and, third, impact effect upon pavements by the Illinois Division of Highways.§

The most complete and satisfactory data on impact in highway bridges are those recently published in *Bulletin 75* by the Engineering Experiment Station of Iowa State College. A general idea of the scope of this work and the results is indicated by the following Synopsis and Conclusions which have been taken from the *Bulletin* and slightly edited.

Preceding the Synopsis and Conclusion, definitions will be given of a few terms which have been used, as follows:

Dynamic force of a wheel blow is the maximum pressure of a truck wheel upon the bridge floor when the truck is in motion.

Impact increment of dynamic force is the additional pressure above the weight of the wheel, due to the fact that the load was in motion.

Impact increment of stress is the additional stress above the static stress due to the fact that the load was in motion.

Stress ratio is the ratio of actual dynamic stress developed in a member to the stress that would have occurred had a static load equal in magnitude to the dynamic force been applied at the same place as the dynamic force was applied.

Sprung weight includes all weight carried by the springs.

Unsprung weight consists of the remaining weight; that is, the wheels, axles, housings, and springs.

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* Presented to the Annual Meeting, January 20, 1926.

† *Proceedings*, Am. Soc. C. E., March, 1923, Papers and Discussions, p. 458; *Public Roads*, September, 1924; *Bulletins 63* and *75*, Eng. Experiment Station, Iowa State College.

‡ *Public Roads*, March and December, 1921; "Researches on the Structural Design of Highways by the United States Bureau of Public Roads," by A. T. Goldbeck, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 264.

§ "Highway Research in Illinois," by Clifford Older, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1180.

Synopsis.—The field covered was the determination of observed stresses due to certain static and dynamic loads (and, therefore, impact) in the steel floor systems and, in some cases, of truss members of five steel bridges with concrete floor-slabs and seven steel bridges with plank flooring.

It has also included the determination of the dynamic force of the wheel blow from several trucks and a relation between the force of the blow and the simultaneous stresses in stringers and floor-beams.

The most significant results are as follows:

1.—Static distribution of stresses between the stringers of four bridges with concrete floor-slabs.

2.—Maximum percentage of the load of one truck wheel which is transferred to one stringer in both concrete and timber floor bridges.

3.—The percentage of impact (impact increment of stress) in the stringers of three bridges with concrete floor-slabs in terms of speed and the percentage of unsprung weights of the trucks. (Figs. 1, 2, and 3.)

4.—The percentage of impact in the stringers, floor-beams, and a few of the truss members of seven bridges with timber flooring.

5.—The ratio of impact increment of force (of wheel blow) to impact increment of stress in stringers and floor-beams for nine bridges, including concrete and timber flooring.

6.—A relationship between the stress ratio and the speed of the truck.

7.—A relationship between the stress ratio and the percentage of the impact increment of the dynamic force. (Figs. 4, 5, and 6.)

Conclusions.—

Static Distribution of Stress in Stringers.—The stringer spacing in the concrete floor bridges under consideration did not vary sufficiently to permit general conclusions. With a stringer spacing varying from 28.5 to 36 in. and with one truck on the span, the maximum observed stresses in one stringer for each of three spans was just about 50% of the total observed stresses for one wheel (25% of the total for the truck). With two trucks on the span the observed stresses for one stringer in terms of observed stresses for one wheel varied from 60% for the 28.5-in. spacing to 69% for the 36-in. spacing.

The observed stresses in all instances are far below the computed ones. This is due to bond between the steel beam and the concrete floor, partial fixity of ends of the stringers, and, perhaps, to other causes.

In three of the four structures the bond seems to be excellent and in all these structures the observed stresses are less than half those which were computed under the usual assumption that the steel stringers act as independent simple beams. In one span, the bond seems to be impaired to a certain extent and the observed stresses are about two-thirds the computed stresses.

Impact in Floor Systems.—The impact increment of stress is due very largely to the unsprung weight of the truck. The percentage of impact, therefore, varies with the total load. It also varies with the flexibility of the floor,

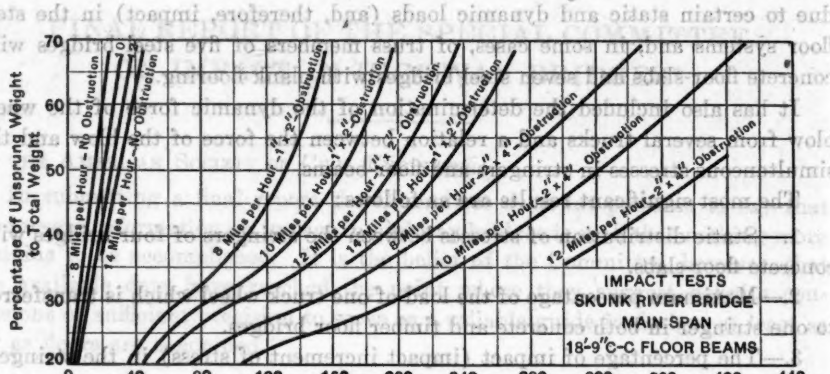


Fig. 1.

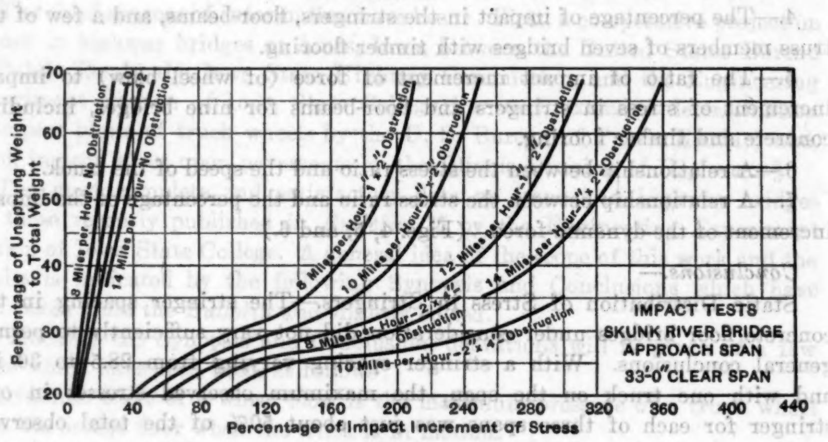


Fig. 2.

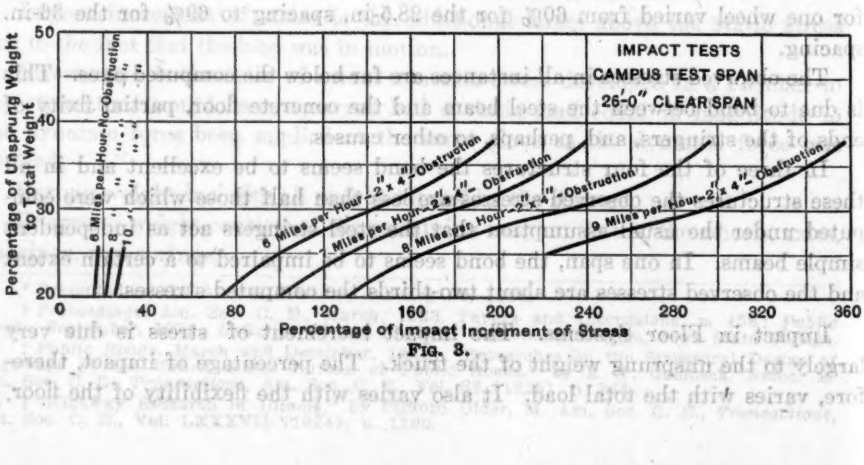


Fig. 3.

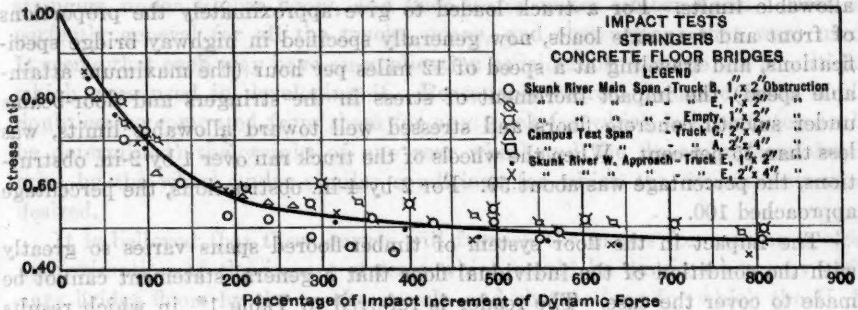


FIG. 4.

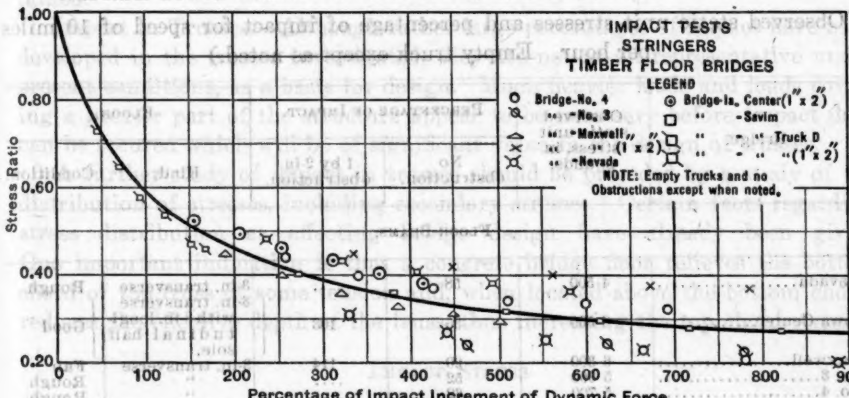


FIG. 5.

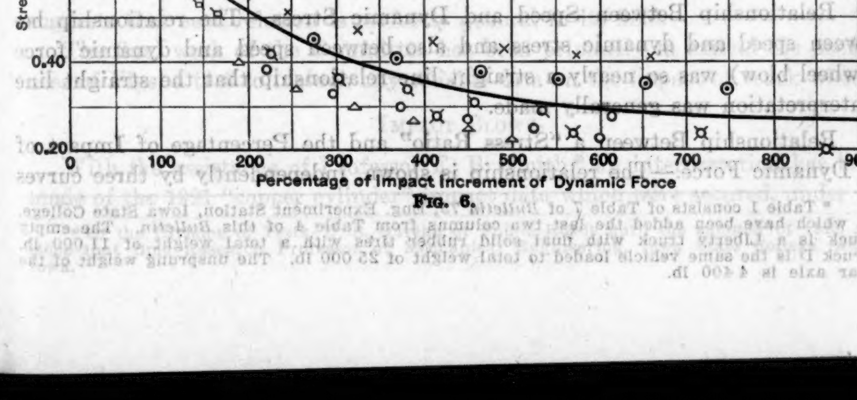


FIG. 6.

being the least in the important cases where the total unit stresses approach allowable limits. For a truck loaded to give approximately the proportions of front and rear axle loads, now generally specified in highway bridge specifications, and traveling at a speed of 12 miles per hour (the maximum attainable speed) the impact increment of stress in the stringers and floor-beams under smooth concrete floors and stressed well toward allowable limits, was less than 15 per cent. When the wheels of the truck ran over 1 by 2-in. obstructions, the percentage was about 50. For 2 by 4-in. obstructions, the percentage approached 100.

The impact in the floor system of timber-floored spans varies so greatly with the condition of the individual floor that a general statement cannot be made to cover the case. The reader is referred to Table 1*, in which results obtained in seven different spans are to be found.

TABLE 1.—IMPACT IN FLOOR SYSTEMS OF TIMBER FLOOR BRIDGES.

(Observed static unit stresses and percentage of impact for speed of 10 miles per hour. Empty truck except as noted.)

Bridge.	Observed static unit stress, in pounds.	Percentage of Impact.		Floor.	
		No obstruction.	1 by 2-in. obstruction.	Kind.	Condition.
Floor-Beams.					
Nevada	4 500	58	....	3-in. transverse	Rough
Iowa Center	6 500	36	103	3-in. transverse with 3-in. longitudinal half sole.	Good
Maxwell	6 300	20	114	3-in. transverse	Fair
No. 3	5 900	52	....	"	Rough
No. 4	5 700	62	....	"	Rough
Savim	3 500	46	....	"	Fair
Savim (Truck D)	9 500	21	76	"	....
Stringers					
Nevada	7 150	75	....	See above	
Iowa Center	8 700	13	109	See above	
Maxwell	5 500	30	217	See above	
No. 3	6 300	121	....	See above	
No 4	3 900	69	....	See above	
Savim	1 850	109	281	See above	
Savim (Truck D)	5 000	22	120	See above	
Cambridge	5 150	40	....	3-in. transverse	Fair

Relationship Between Speed and Dynamic Stress.—The relationship between speed and dynamic stress and also between speed and dynamic force (wheel blow) was so nearly a straight line relationship that the straight line interpretation was generally made.

Relationship Between a "Stress Ratio" and the Percentage of Impact of a Dynamic Force.—The relationship is shown, independently by three curves

* Table 1 consists of Table 7 of *Bulletin 75*, Eng. Experiment Station, Iowa State College, to which have been added the last two columns from Table 4 of this *Bulletin*. The empty truck is a Liberty truck with dual solid rubber tires with a total weight of 11 000 lb. Truck D is the same vehicle loaded to total weight of 25 000 lb. The unsprung weight of the rear axle is 4 400 lb.

(Figs. 4, 5, and 6), one each for steel stringers under concrete floors, steel stringers under timber floors, and floor-beams under timber floors. Each is perfectly general for all the trucks, spans, and obstructions that were used. It seems that each may have an application considerably outside the conditions which were used in developing it. Reasonably correct dynamic stress could doubtless be computed from them for any truck for which is known (or could be determined) the weight of one rear wheel and the dynamic force developed by the wheel under similar conditions for which the dynamic stress is desired.

It is believed that these curves will furnish the means for making a close approximation of the impact stresses which would be developed in any ordinary bridge floors by the trucks, speeds, and obstructions for which the U. S. Bureau of Public Roads has already determined the dynamic force of the wheel blow; and also for any other cases for which similar data may be obtained.

Impact in Trusses.—Although rather large percentages of impact have been developed in the trusses investigated, they are not at all representative under present conditions, as a basis for design. Much heavier loads and loads covering a greater part of the structure appear to be necessary before impact data can be secured which will be of significant value in the design of trusses.

A further study of impact in trusses should be preceded by a study of the distribution of stresses, including secondary stresses. Certain facts regarding stress distribution as affecting bridge design have already been given. One important indication is that a concrete bridge floor relieves the bottom chord of the truss of some tension and, when located above the bottom chord, reduces the effective depth of the truss, thus increasing the top chord stresses.

Impact Stress

Figs. 1 to 3 taken from *Bulletin 75*, previously mentioned, give, for the stringers of three bridges with concrete floors, the percentage of impact increment of stress in terms of percentage of unsprung weight to total weight, the speed of the trucks, and obstructions on the floor.

Figs. 4 to 6 from the same *Bulletin* show impact increment of (or increase in) dynamic force platted against the stress ratio.

These three diagrams—one each for steel stringers under concrete floors, steel stringers under timber floors, and floor-beams under timber floors—indicate a relation between impact increment of dynamic force and the stress ratio for a variety of obstructions, loads, and spans. The available information suggests that each relation is perfectly general in its field.

The final work of the Committee has been primarily to extend the usefulness of Figs. 4 to 6 to include any reliable data on the impact of truck wheels.

Impact Blows

With the assistance of Professor E. B. Smith,* an interpretation has been made of the 1921 "copper cylinder" impact data which were secured, under his

* Of the Division of Tests, U. S. Bureau of Public Roads until the summer of 1925 and since that time Research Professor of Mechanical Engineering, Iowa State College, Ames, Iowa.

direction, by the U. S. Bureau of Public Roads, along lines suggested by him in his paper* descriptive of those results. This interpretation has been made more readily and more clearly by comparison with similar data taken from *Bulletin 75* of the Engineering Experiment Station of Iowa State College.

All available data on the force of the blows of truck wheels have been platted in terms of the unsprung weight, the flexibility of the tires, and the dynamic force of wheel blows. Curves have been drawn through the platted points and, with slight modifications, have been made to conform to reasonably simple equations.

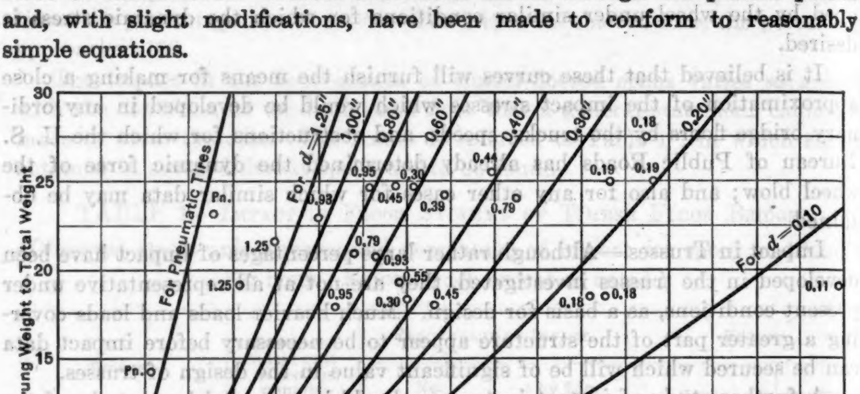


FIG. 7.

One of six diagrams is reproduced as Fig. 7. The equations of the curves are $p = a I^{1.6}$, in which, p is the percentage of unsprung weight to total weight and I is the impact blow, in percentage. The relationship between the values of a of the various curves of Fig. 7 is $a = 0.0069\ (d)^{0.72}$, in which, d is the tire deformation, in inches, due to a static load of 10 000 lb. The values of a vary differently for the other five diagrams, which include speeds of 12, 15, and 18 miles per hour and 1-in. and 2-in. obstructions.

The relationship for all the conditions which were investigated may be expressed by the equation:

$$I = \sqrt[1.6]{\frac{p \cdot H^{1.555}\ S^{1.5}}{0.29\ d^{0.72}}} \qquad (1)$$

in which,

I = impact increment in percentage of dynamic force of wheel blow.

p = unsprung weight in percentage of total weight.

* *Public Roads*, March and December, 1921.

H = height, in inches, of obstruction on bridge floor. (Equals about 0.16 for the bare concrete floors and an average of the best timber floors for which data are available.)

S = speed of truck, in miles per hour.

d = tire deformation, in inches, under a static load of 10 000 lb.

Equation (1) has been platted for 1-in. and for 2-in. obstructions on Fig. 8, and for 0.16-in. obstruction, representing bare floors, on Fig. 9.

In these diagrams the parabolas, $p = a\,I^{1.6}$, were plotted at various values of a, covering the range of trucks likely to be used, and tires ranging from very hard dual types having deformation of only 0.1 in. due to a 10 000-lb. load, to pneumatics. The cross-curves were plotted from the equations:

$$a = 0.29\ (s)^{-1.5}\ (d)^{0.72}, \text{ for 1-in. obstructions;}$$
$$a = 0.099\,(s)^{-1.5}\ (d)^{0.72}, \text{ for 2-in. obstructions; and}$$
$$a = 3.524\,(s)^{-1.5}\ (d)^{0.72}, \text{ for bare floors.}$$

Equation (1) seems to be perfectly general for determining the impact of blows delivered to concrete and to timber bridge floors. The diagram in Fig. 8 was first constructed for concrete floors. The 1-in. obstruction curves give correct results for 1-in. obstructions on the smoothest timber floors and results which are reasonably close to observed values without obstruction on the roughest timber floors.

The diagram in Fig. 9 was constructed for the height of obstruction (0.16 in.) which most nearly represented the conditions of the bare concrete floors. It applies fairly well to an average of the better timber floors. For the smoothest timber floor which was available (longitudinal plank in good condition laid over transverse plank) the percentages of impact blow were about 15 to 20 less than those given in Fig. 9. The results for this floor may be closely approximated by a value of H in the formula of $\frac{1}{8}$ in.

The rougher timber floors naturally produce greater impact. Available data show impact on rough timber floors without other obstructions to be fully as great as for 1-in. obstructions on smooth floors.

Since developing Equation (1) and the three supplementary ones for a and constructing Figs. 8 and 9, Equation (1) has been slightly modified and considerably simplified by putting it into the following form:

$$I = \frac{1.8\,H\,S\,(p)^{0.625}}{(d)^{0.45}} \quad\quad (2)$$

Numerous checks which have been made by comparing results from Equation (2) with those from Equation (1) and Figs. 8 and 9, have disclosed no differences as large as 3 per cent. As far as can be determined, Equation (2) represents the mass of experimental data about as well as Equation (1). It would seem that either of the equations or Figs. 8 and 9 may be used with confidence as representing, within necessary precision, the impact in highway floors.

The new "cross-curve" equations for a to correspond with Equation (2) are:

For 1-in. obstructions:

$$a = 0.39\ S^{-1.6}\ d^{0.72}$$

For 2-in. obstructions:

$$a = 0.129\ S^{-1.6}\ d^{0.72}$$

For bare floors (0.22-in. obstructions):

$$a = 4.50\ S^{-1.6}\ d^{0.72}$$

The parabolas would be platted, as before, from the equation, $p = a\ I^{1.6}$, using values of a as given previously.

In using Equation (2), the corresponding height of obstruction for the smoothest of the timber floors becomes 0.18 in.

The use of these diagrams is indicated by the following illustrative problem.

Given a truck with tire deformation of 0.40 in. due to a static load of 10 000 lb. and a total one rear-wheel load of 6 000 lb., of which 2 100 lb., or 35%, is unsprung, obtain the blow of this wheel due to a run over a 1-in. obstruction on a concrete floor bridge at a speed of 15 miles per hour.

Beginning at the right side of the diagram (Fig. 8), at the speed of 15 miles per hour, go to the left, parallel to the x-axis, until the solid-line short curve marked 0.40, tire deformation, is intercepted. Then go upward, parallel to the y-axis, to the top of the diagram where the a-values are given. Then follow the parabola downward toward the origin, interpolating between the given parabolas, until the percentage of unsprung weight, 35, is intercepted. Then go down from this point, parallel to the y-axis, to the percentage impact blow, which, in this case, is 385.

The actual blow which the wheel can deliver under these conditions then is:

$$6\,000 + 6\,000\ (3.85) = 29\,100 \text{ lb.}$$

Use of Impact Blows in Determining Impact Stresses

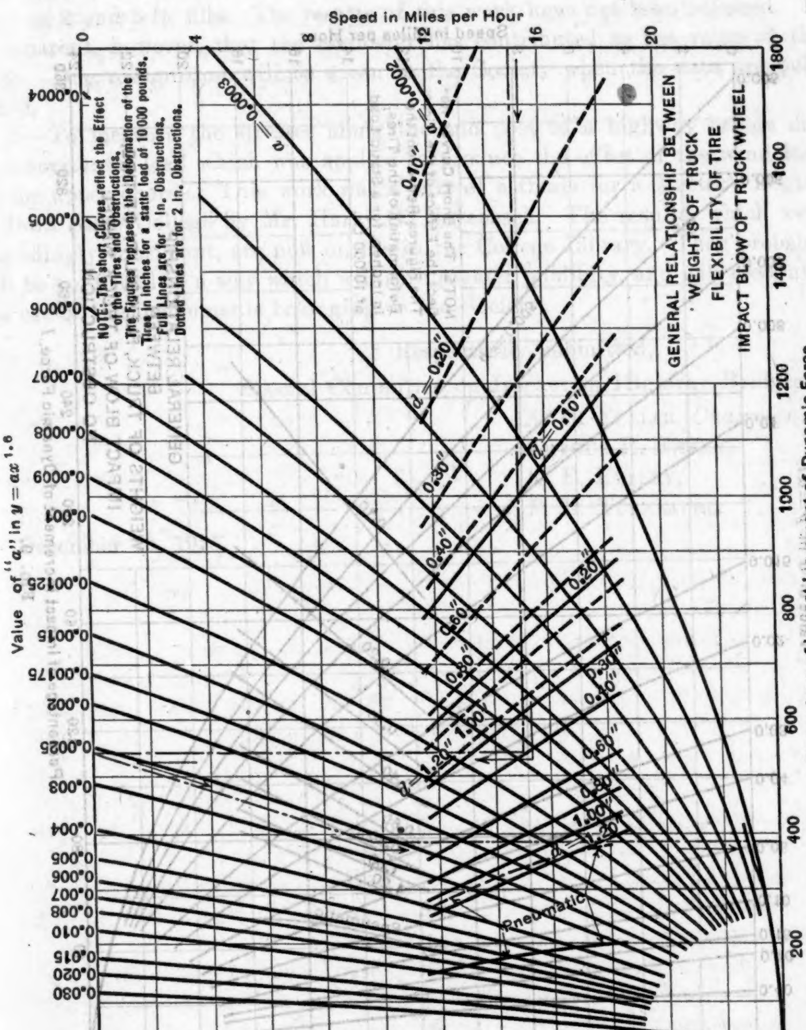
Knowing the percentage impact blow, sprung and unsprung load, and static stress due to the load in the stringers, it is possible by the judicious use of other curves (Figs. 4, 5, or 6) developed by the Iowa Engineering Experiment Station, to determine the actual stringer stress within the necessary precision.

This illustrative example may be continued by entering Fig. 4 with 385% impact increment of dynamic force and reading a stress ratio of 0.52. The stress, then, in a stringer of a bridge with a concrete floor, due to the dynamic force of the 6 000-lb. wheel load, will be 0.52 times the stress due to a static load of 29 100 lb.

Use of the Electric Telemeter

The McCollum-Peters electric telemeter which was made for the Society by the U. S. Bureau of Standards for the use of the Committee was assigned again in 1925 to the co-operative project at Ames, Iowa, which has been previously mentioned.

The Committee has also authorized its use for brief periods on two other projects:



GENERAL RELATIONSHIP BETWEEN
WEIGHTS OF TRUCK
FLEXIBILITY OF TIRE
IMPACT BLOW OF TRUCK WHEEL
NOTE: The short Curves reflect the Effect of the Tires and Obstructions.
The Figures represent the Deformation of the Tires in Inches for a static load of 10 000 pounds.
Full Lines are for 1 In. Obstructions.
Dotted Lines are for 2 In. Obstructions.
Speed in Miles per Hour
Value of "a" in $y = ax^{1.6}$
Percentage of Unsprung Weight to Total Weight
Percentage of Impact Increment of Dynamic Force
Pneumatic

FIG. 8.

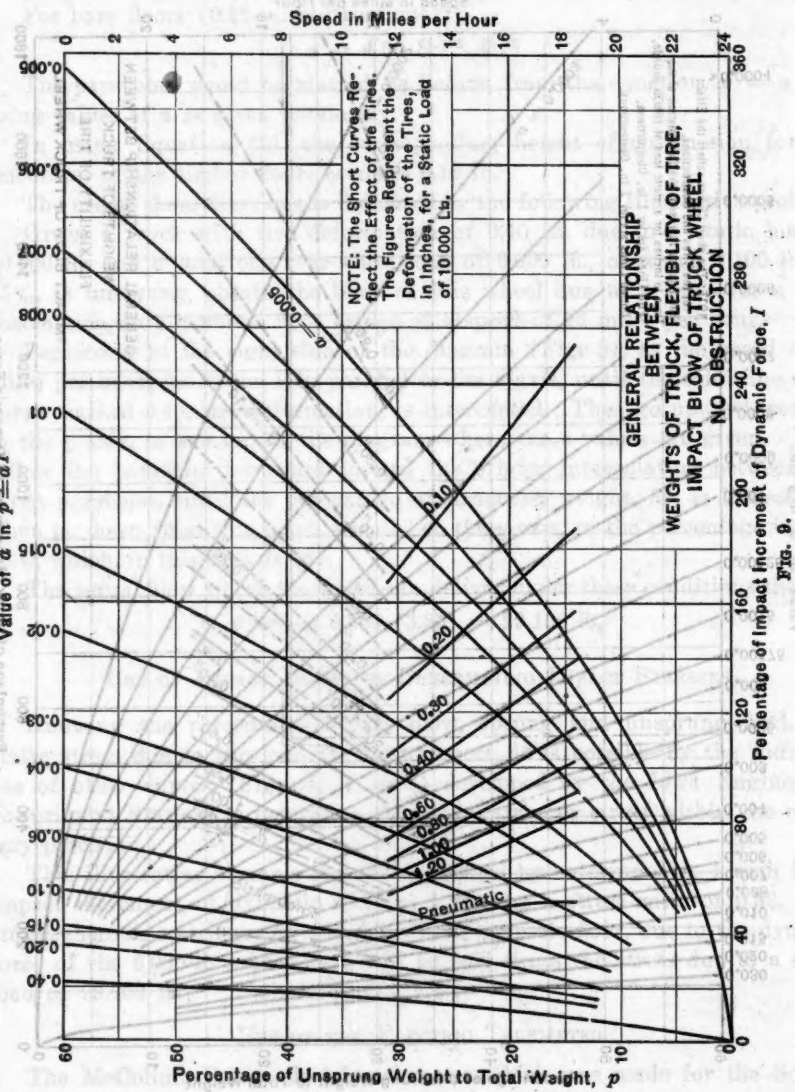

Speed in Miles per Hour
0
2
4
6
8
10
12
14
16
18
20
22
24
NOTE: The Short Curves Reflect the Effect of the Tires. The Figures Represent the Deformation of the Tires, in Inches, for a Static Load of 10 000 Lb.
GENERAL RELATIONSHIP BETWEEN WEIGHTS OF TRUCK, FLEXIBILITY OF TIRE, IMPACT BLOW OF TRUCK WHEEL NO OBSTRUCTION
a = 0.005
0.10
0.20
0.30
0.40
0.60
0.80
1.00
1.20
Pneumatic
Value of a in p = a I^1.8
0.005
0.006
0.007
0.008
0.01
0.015
0.02
0.03
0.04
0.06
0.10
0.15
0.20
0.40
Percentage of Unsprung Weight to Total Weight, p
60
50
40
30
20
10
0
Percentage of Impact Increment of Dynamic Force, I
0
40
80
120
160
200
240
280
320
360

Fig. 9.

1.—As a means of contributing to the impact data on a co-operative culvert project between the U. S. Bureau of Public Roads and the Engineering Experiment Station of Iowa State College. The impact work on this project has been done on earth fills over the culvert from 2 to 5 ft. The telemeter was used on 2 and 3-ft. fills. The results of this work have not been released. It is apparent, however, that the telemeter has contributed to the value of the work. Due recognition will be given to the Society when the data are published.

2.—To measure the stresses along the end post of a highway bridge due to a horizontal load which was applied to produce the effect of the wind load on the upper chords. This work was a part of a thesis for a Master's Degree at Iowa State College by Mr. Harry G. Neyenesch. The results, which were exceedingly consistent, are now on file in the College Library. They probably will be published in a way which will give greater publicity, and will recognize the use of the instruments belonging to the Society.

Respectfully submitted,

Special Committee on Impact in Highway Bridges.

A. H. FULLER, *Chairman,*
ARTHUR R. EITZEN,
E. F. KELLEY,
F. E. TURNEAURE.

December 19, 1925.

THE REINFORCED CONCRETE ARCH IN SEWER CONSTRUCTION: A REVIEW OF PAST PRACTICE IN DESIGN AND AN ACCOUNT OF RECENT STUDIES IN ST. LOUIS, MISSOURI

Discussion*

By Charles E. Sharp, Jr., Esq.†

Charles E. Sharp, Jr.,‡ Esq. (by letter).§—The writer desires to express his appreciation of the many helpful criticisms of the paper, and to state his agreement with the thought which carries through the discussion, namely, that earth pressure is a very uncertain quantity. The suggestion that models of a sewer barrel be made and subjected to certain typical loadings is apt. However, the writer feels that only a volume of "missionary work" will effect the desired result, that of convincing those responsible for carrying out a large project that such experiments may be well worth the expense involved.

The fundamental difficulty is in the profession itself. That the civil engineer, in large measure, fails to receive the proper confidence and respect of his financial superiors is evident. That many an engineer is treated as an artisan, is paid less than an artisan's wages, and has little encouragement for creative work is also evident. For, in the eyes of the world, any schoolboy can come into a drafting-room, and design a satisfactory beam, hiding his ignorance or lack of experience behind the safety factor of 4 or maybe 10. There is no apparent mystery about the profession as in law, medicine, etc. If the average engineer could talk about beams, columns, etc., in Latin or Greek terms, he might command more awe, more money, and then he might order an earth pressure experiment with the same assurance as a specialist in the Medical Profession orders his patient to take "Pythagorean" pills for a common headache.

As for the actual horizontal pressures used, the writer feels that when they are combined with water pressure, the net relief gained for the vertical loading is well within reason; that even if this relief were reduced arbitrarily to zero, the stresses throughout the arch ring would not increase more than, say, 25 per cent. Mr. Whitney's discussion‖ illustrates rather than contrasts the writer's thought on this adjustment of external and internal pressures. In

* Discussion on the paper by Charles E. Sharp, Jr., Esq., continued from February, 1926, *Proceedings*.

† Author's closure.

‡ Asst. Engr., Mo. Pac. Ry., St. Louis, Mo.

§ Received by the Secretary, January 26, 1926.

‖ *Proceedings*, Am. Soc. C. E., February, 1926, Papers and Discussions, p. 278.

Fig. 15,* for Loadings *b* and *c* the bulging of the sides of the arch due to water pressure is counteracted by the earth pressure set up as a reaction to the bulging.

The writer desires to state most emphatically that the major economy of the River Des Peres design was not realized by considerations of horizontal earth pressure, but by assuming the soil reaction to be concentrated within the region of the spread footings. As this assumption was opposed vigorously by Mr. Terzaghi,† the writer desires to clarify his opinion as to what happens at the bottom of a sewer arch.

Mr. Terzaghi is right in stating that the invert arch, if confined between unyielding abutments, is stiff enough to take its part of the reactive load, and, therefore, would exert a pressure which the upper arch is wholly incapable of absorbing. Apparently, he feels, since "the joints of invert and footing are trifling construction details, and do not affect fundamentally the distribution of bending moments", that the invert and arch act together as a monolithic ring, and thus actually form the same structure as the conventional type of sewer arch. Naturally, he concludes that the writer's thin invert will have a greatly reduced factor of safety, since the monolithic ring with soil pressure over the entire bottom requires an invert of considerable thickness.

Now, two considerations would tend to upset this calculation and to justify the writer's assumption of a structure truly separated at the joints in question:

First.—In order for the structure to act as a monolithic ring the span length must increase. Take the case of a constant span (disallowing the writer's rotating hinge action), the invert arch will contain throughout its length a heavy horizontal thrust. The monolithic ring contains no such thrust, and the "difference", so to speak, is dissipated by the expansion of the invert. This forced increase in span, however, is resisted in ordinary clay bearings by friction of the sewer against the soil, and also by the horizontal pressures of the soil against the cup-shaped invert.

Second.—Although the writer maintains that, for practical purposes, the span or distance between centers of footings remains constant, this assumption does not govern the footing at its edge or at its junction with the invert arch. As stated by the writer‡ and illustrated by Fig. 15(*a*), the spread footing under vertical loading tends to rotate around its own center point, outer edge down, and thus to lift and slightly lengthen the invert arch, and to relieve the latter of any reactive load that may have been assigned to it. The analysis of the hinged arch with only vertical loading shows a certain horizontal thrust at the crown, and, therefore, a negative thrust or pull of the footings on the ends of the invert. This "pigeon-toed" action of the arch footings results from the shouldering or bulging of the arch sides. It is important to note here that this very bulging indicates the arch to be highly stressed throughout its length. Horizontal earth pressure opposes these conditions, but, being

* *Proceedings*, Am. Soc. C. E., August, 1925, Papers and Discussions, p. 1112.
† *Loc. cit.*, December, 1925, Papers and Discussions, p. 1985.
‡ *Loc. cit.*, August, 1925, Papers and Discussions, p. 1086.

correctly taken (for safety) as a minor force, it will have a minor effect in diminishing the lifting of the invert.

In summary, it appears that the very loads which highly stress the arch tend to relieve the invert of reaction, thus confining the reaction to the spread footings. The case of a heavy horizontal earth pressure with small vertical load could hardly upset this consideration, inasmuch as the soil load in this case would be necessarily low, and could not cause critical stresses even if it were considered as uniformly distributed over the invert. In view of these considerations it seems logical to assume the invert arch to be entirely capable of taking any small horizontal thrust that may result from horizontal earth pressure, since the arm of the thrust about the center line of the invert is small also. It was the writer's intention to allow the invert to act as a curved beam with reinforcing steel near the bottom surface.

Mr. Terzaghi is right in objecting to the projecting parts of the spread footings in that they might allow the formation of water-pockets just above them. In designing the Blue River and Gooseneck Creek Sewers in Kansas City, Mo., recently, the writer's attention was called to this same possibility. As a result, tile drains were used at intervals of 25 ft., placed across the arch rib just above the tops of the footing, and allowed to slope down into the sewer.

MULTIPLE-ARCH DAM AT GEM LAKE ON RUSH CREEK, CALIFORNIA

Discussion*

By Messrs. F. W. Scheidenhelm, Charles W. Comstock, and George W. Howson.

F. W. Scheidenhelm,† M. Am. Soc. C. E.—The discussion‡ by Senator Luiggi is of exceeding interest to the speaker.

Senator Luiggi is of the land which presumably was the first to develop cement, namely, the Roman or puzzolan cements. From much that has been said and written (not by Senator Luiggi) one would infer that Roman cements, or more particularly the structures in which they were used, are everlasting. If so, an important art has been lost. However, the speaker suspects that the long life actually enjoyed by certain Roman puzzolan cement-bound structures is due to the fact that, except for the northern part, Italy is not subject to a rigorous climate. Freezing either does not occur or is of minor extent. In fact, the speaker's personal observation on the occasions of two visits to Italy lead him to believe that, under similar climatic conditions, Roman cement-bound structures behaved much like Portland cement-bound structures of the present age. In neither case does the cement seem to be perfect. Even Italy, as well as America, has its problems as to the durability of hydraulic structures.

A Fundamental Question

The Gem Lake Dam is a hollow concrete structure which, by reason of disintegration of its deck, has failed to perform its intended function. Long before the presentation of this interesting paper there was raised in the speaker's mind the fundamental question: Is a hollow dam of concrete an appropriate engineering structure, particularly for cold climates? This question carries with it the premise that the concrete is of a kind which is practicable of attainment under the present state of the art. The question applies to hollow concrete dams of both the multiple-arch and the reinforced concrete slab-deck types. Study of this question has led the speaker into a fairly broad investigation and the present seems an opportune occasion for briefly stating the results and presenting his deductions.

At the outset it may be stated broadly that the investigation has left the speaker optimistic as to the propriety of the use of concrete in hydraulic

* Discussion of the paper by Fred O. Dolson and Walter L. Huber, Members, Am. Soc. C. E., continued from February, 1926, *Proceedings*.

† Cons. Engr. (Mead & Scheidenhelm), New York, N. Y.

‡ *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 90.

structures which are exposed to severe freezing and thawing and, until there is further and contrary evidence available, his answer to the foregoing question is definitely, even though qualifiedly, in the affirmative.

Gem Lake Dam Disintegration

First consider briefly the conditions obtaining in the case of the Gem Lake Dam. The design involves relatively thin arches. The thickness was apparently from about 16 to about 34 in. within the areas of the arched decks where disintegration occurred. However, relatively thin decks, whether in the form of arches or reinforced concrete flat slabs, constitute an essential feature of hollow concrete dams. If the decks were not relatively thin, then hollow concrete dams would lose a considerable proportion of any economy which they may offer. It is fair to state that the design of the Gem Lake Dam follows the general lines accepted as structurally satisfactory for dams of its type.

As to the concrete of which the dam was originally built, the facts available from published information and private inquiry are none too detailed nor reassuring. Among other things it is not unlikely that the sand was considerably too fine. The coarse aggregate is questioned by certain interested engineers, but the information available is not such as to cause the speaker to attach any serious blame to it. The cement was presumably of a quality equal to the average of standard Portland cements and subject only to the qualifications that there is still much to be learned about cement in general and that undoubtedly there are some steps still to be taken for the improvement of the product.

The proportions in which the arch concrete is reported to have been mixed, namely, 1 part of cement to 6 of aggregate, might be adequate if the other factors of the manufacture of the concrete in point were satisfactory. However, it seems not unlikely that the fineness of the sand would have warranted a somewhat greater cement content. Moreover, a greater cement content would have been desirable, if indeed not imperative, in case the water-cement ratio was relatively high. As to the latter, the speaker does not doubt that in conformity with general practice at the time of the construction of the Gem Lake Dam (1916) the water content was too great. (If the builders of the Gem Lake Dam erred in this respect, let it be noted that many others, including the speaker, have made the same error.)

The speaker has been unable to learn that any particular attention was paid to the adequate curing of the concrete and agrees thoroughly with Mr. Merriman* that conditions were most unfavorable for proper curing. Although impermeability and permanence of concrete are not direct functions of strength, nevertheless, it is pertinent to note that the 14-day strength of specimens of concrete tested during the original construction was only about 900 lb. per sq. in. in compression. That fact in itself was none too good an omen.

The outstanding characteristic of the disintegration of the concrete is that apparently the arch mortar has tended to change from the original texture and strength to a texture and strength approximating those of hard clay, or per-

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 94.

haps even chalk. The disintegration is reported to have been limited to the deck arches, the buttresses remaining in good condition. Although presumably there was a tendency to require greater workability and, hence, a wetter mix for the arch concrete than for the buttresses, yet this possibility and the disintegration of the arch do not necessarily indicate that the arch concrete was inferior to the buttress concrete—in fact, the buttress concrete is reported to have had a somewhat lower cement content. It seems clear only that the arch concrete was not of sufficiently good quality to withstand the severe physical conditions to which it was subjected. It is likely that if the buttress concrete were subjected to similar physical conditions for an equal length of time similar disintegration would result.

As pertinent to the causes of the disintegration and the possible means for preventing similar disintegration in other structures, it is to be noted that apparently the disintegration occurred only below the winter water surface of the reservoir. The reservoir is reported to have been drawn down regularly for the winter period to a level about 30 ft. lower than the "pool-full" level. Correspondingly, the upper limit of the disintegration is said to have been about 30 ft. below the top. It is, therefore, a reasonable deduction that percolating water from the reservoir was a prerequisite of the disintegration.

On the other hand, the authors report* that "a section of the bottom of each arch was little affected." Clearly the absence of disintegration in the lower part of the arched decks cannot be attributed to the absence of percolating water. It is true that the lower part of each arched deck is thicker than that directly above, but such a difference in thickness is too gradual and too minor in amount to afford the explanation. Instead the speaker is strongly inclined to agree with the authors in attributing that better condition to the protection afforded by the drifts of snow which lodge against the lower parts of the decks.† Precipitation appears to be plentiful and must be in the form of snow even before the temperature becomes very low.

The absence of disintegration in the lower parts of the decks indicates that percolation of reservoir water through the concrete is not necessarily, or at least not rapidly, harmful provided there is no severe freezing at the under side of the deck. (Whether in the case of even the best quality of concrete permitting percolation of water such percolation might not *per se* cause some disintegration in the course of a number of years is another question.)

The speaker inclines to the view that the disintegration of the arched decks at Gem Lake was due to the combination of inferior concrete, percolating water, and severe freezing alternating with thawing, and that, in the absence of any one of these three unfavorable factors, disintegration would probably not have occurred. In fact, the number of unfavorable factors may practically be reduced to two, for if the concrete were impermeable percolation could not have occurred. It is of special significance that the down-stream faces of those parts of the arched decks which suffered damage were at no

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1320.

† Charles W. Comstock, M. Am. Soc. C. E., has informed the speaker that he was told by members of the local operating staff of the Power Company that the near-by 30-ft. Agnew Lake Dam becomes completely covered with snow on its down-stream side.

time of the year protected by structural or natural (snow) enclosure against severe freezing.

Successful Unenclosed Hollow Concrete Dams

Engineers may well take cognizance of the fact that there are in existence a number of hollow concrete dams, both of the multiple-arch and the reinforced concrete slab-deck types, which, despite their locations in regions of rigorous climate involving alternate freezing and thawing, have continued to perform their intended functions and which have suffered no disintegration or, if there has been any disintegration, such disintegration has been too insignificant to record. This fact indicates strongly that such difficulties as have been encountered with hollow concrete dams have not been due primarily to the thin decks, whether in the form of arches or of reinforced concrete plates. Also, it shows that concrete can be produced which, in the accepted sense, is permanent and can successfully withstand the rigors of such climates.

Agnew Lake Dam, California.—Of existing hollow concrete dams which are performing their intended functions the two which have the most direct bearing on the Gem Lake case are the Agnew Lake Dam (the mate of the Gem Lake Dam) and the Lake Eleanor Dam. The Agnew Lake Dam was built by the same company, by the same construction organization, and at the same time as the Gem Lake structure, so that it, too, has had a life of more than nine years.* It is situated on the same stream, at an elevation only about 500 ft. lower; hence, both dams are subject to practically the same climatic conditions. The Agnew Lake Dam, however, is only 30 ft. high and is in a small gorge, so that its crest length is only 280 ft. In consequence, the dam is reported to be snowed in every winter, with the snow on the downstream side extending to the very top of the structure. L. R. Jorgensen, M. Am. Soc. C. E., the designer of both dams, states that Agnew Reservoir remains practically full throughout the winter.

The design of the Agnew Lake Dam is the same as that of the Gem Lake Dam. In view of the low height of the Agnew Lake Dam, therefore, the thicknesses of arch decks subject to hydrostatic pressure from the reservoir during the winter are nowhere so great as in the cases of those decks of the Gem Lake Dam which have suffered disintegration.

The concrete of the Agnew Lake structure is practically the same as that at Gem Lake. The coarse aggregate was taken from Gem Lake to Agnew Lake. The sand apparently involved the only difference from the Gem Lake construction, as about one-half of it came from Gem Lake and the remainder from Agnew Lake itself. Hence in spots, at least, even the sand must have been the same as at Gem Lake; on the other hand, Mr. Jorgensen was of the impression that the local Agnew Lake sand was of the poorer quality. As regards water and cement contents and the method of making the concrete, the reasonable assumption is that there was no difference between the two structures.

* "Multiple-Arch Dams on Rush Creek, California," L. R. Jorgensen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 850.

Nevertheless, competent engineer observers agree that the Agnew Lake Dam is in first-class condition, having suffered no disintegration. Why, then, the difference? Presumably the unfavorable conditions as regards quality of concrete, percolation of water, and severity of climate are essentially the same. The one marked difference appears to be that involved in the complete snow covering, thus obviating severe freezing. The speaker agrees with Mr. Jorgensen in attributing the good condition of the Agnew Lake Dam to its snow cover protecting the down-stream faces of the arched decks.

Lake Eleanor Dam, California.—The Lake Eleanor Dam, of the City of San Francisco, located on a tributary of the Tuolumne River, is likewise of the multiple-arch type and is of approximately the same height, namely, 70 ft. gross against the 80 ft. gross height of the Gem Lake Dam. Lake Eleanor Dam is on the western slope of the Sierra Nevadas at an altitude of only about 4 650 ft. above sea level, whereas the Gem Lake and Agnew Lake Dams are on the eastern slope of the same mountains at elevations about 4 000 ft. higher. Nevertheless, the Lake Eleanor Dam, too, is subject to the most important and severe element of climatological conditions—alternate freezing and thawing; in fact, the conditions at the Lake Eleanor site are, not unlikely, more severe in that, by reason of the lower altitude, the cycle of freezing and thawing may occur more frequently within a given time.

The panels of Lake Eleanor Dam between centers of buttresses are 40 ft., just as in the case of Gem Lake. Its length is approximately 1 260 ft. It was built in 1917-18 and the same engineer had an important part in the design of each of the two structures. Although there are certain differences in the thickness of members, these are relatively minor; it is fair to state that structurally the two dams are essentially alike. Furthermore, the likeness extends to the conditions of operation, for in each case the water surface of the reservoir is reported to be comparatively low in winter. Possibly there is somewhat more fluctuation of water surface during the winter season in the Lake Eleanor Reservoir than in the Gem Lake structure. As in the case of the latter, the Lake Eleanor Dam has no spillway apron, nor any other type of enclosure of the space between buttresses. Its spillway is in a solid gravity part of the dam.

That there has been no disintegration has been stated* by M. M. O'Shaughnessy, M. Am. Soc. C. E., according to whose report, the present multiple-arch dam at the Lake Eleanor site is in due course to be displaced by a rock-fill dam.† The ultimate dam is to be between 150 and 175 ft. high.‡ Apparently, the designers consider a multiple-arch dam—or a buttressed arch dam, in the terminology which they properly prefer—as being either unsafe or uneconomical for so great a height as that ultimately intended for the Lake Eleanor site. Nevertheless, the arched decks of the existing dam are intended to be utilized and relied on to furnish the lower 70 ft. of water-tight up-stream diaphragm for the ultimate, high, rock-fill dam. Evidently, those in responsible charge entertain no doubts as to the durability of this particular

* *Engineering News-Record*, July 30, 1925, p. 194.

† *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 98.

‡ *Engineering News-Record*, September 4, 1919, p. 466.

multiple-arch dam. Mr. O'Shaughnessy characterizes the Lake Eleanor Dam as being a structural, as well as an economic, success.*

Aziscohos Dam, Maine.—A successful unenclosed hollow concrete dam which has had a fairly long life is the Aziscohos Dam, built in 1910-11, on one of the tributaries of the Androscoggin River in Maine. It is about 75 ft. high and the hollow concrete part is about 500 ft. long, of which one-half is spillway. The spillway portion has an apron extending down about two-thirds the way from the spillway crest to the foundation rock, but not a sufficient distance to have a material effect on temperature conditions in that part of the dam as compared with the bulkhead part, where there is not even a partial enclosure on the down-stream side.

The deck is reinforced and has a flat surface on the up-stream side, but is arched only on the down-stream side. The minimum thickness of the deck slab at the crown of the arches is apparently about 30 in. and the maximum about 36 in. The deck concrete is said to be of 1 : 2.5 : 3 proportions. A detailed description of the dam has been published.†

On the occasion of an inspection of the Aziscohos Dam in June, 1922, the only disintegration observed by the speaker was in the cases of two reinforced concrete brace-beams in the outlet bay, exposed to water and freezing. This good condition is confirmed by the observation of George C. Danforth, M. Am. Soc. C. E., who visited the structure in 1923. Admittedly twelve years is not a long life for a dam; on the other hand, in practically every case of serious disintegration of concrete the trouble has made itself known within ten years.

Eugenia Falls Development Dam, Ontario, Canada.—A hollow concrete dam exposed to temperature conditions approximating those of Gem Lake Dam is that of the Eugenia Falls development of the Hydro-Electric Power Commission of Ontario. This dam is of the Ambursen type, with the conventional reinforced concrete flat deck slab. It is situated about 70 miles northwest of Toronto, Ont., Canada, in latitude 44° 20′ North, where it is subject to a maximum annual variation of temperature of about 125° Fahr., ranging from 35° below zero to 90° above. The structure was completed in 1914, and thus has been in service for more than 11 years. It is about 50 ft. high and the hollow concrete part is about 1 250 ft. in length.

This part of the structure is entirely of bulkhead section, that is, open on the down-stream side, except a relatively short spillway the apron of which extends so short a distance below the spillway crest as to have negligible enclosing effect. During the winter the reservoir surface fluctuates throughout the upper one-third of the height of the dam. In this part of the structure the reinforced concrete deck slabs are about 16 in. thick. The deck concrete is of 1 : 2 : 4 proportion, the minimum thickness being 12 in.

In the main the foregoing information regarding the Eugenia Falls Dam was furnished by the Chief Engineer of the Commission, F. A. Gaby, M. Am. Soc. C. E. Dr. Gaby has further stated that recent inspection showed that the lower faces of the deck slabs have not disintegrated. A minor amount of flaking is said to be in evidence in the upper part of the up-stream face of the

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 95.

† *Engineering News*, March 9, 1911, pp. 288–291.

deck at some of the construction joints, this being attributed to the fact that the upper parts of many of the deck slabs were poured during the unfavorable temperatures of December. From the performance of this structure Dr. Gaby infers that concrete with proper proportioning and good mixing, so as to result in a dense product, will successfully withstand rigorous conditions of cold.

Enclosed Hollow Concrete Dams

One need not be surprised that there are various instances of hollow concrete dams (generally of the Ambursen type) which are completely enclosed along the down-stream faces of the buttresses and which have suffered no disintegration of concrete. In general, the enclosure results from the use of a complete spillway apron. An example of such structures, which has been visited in person by the speaker, is the Warrior Ridge Dam of the Penn Central Light and Power Company, on the Frankstown Branch of the Juniata River in Pennsylvania. This structure, about 30 ft. high and about 450 ft. long, is entirely of spillway section. It was constructed in 1906, thus having about 20 years service to its credit. The speaker's earlier observation as to absence of disintegration has been confirmed more recently by several other engineers.

Other Cases of Disintegration

To mention several other instructive cases where disintegration of concrete in dams has occurred, the first is an intake dam,* on Bishop Creek, in California, likewise on the eastern slope of the Sierra Nevadas, which suffered some disintegration on the down-stream face of the relatively thin section of the dam. This disintegration is reported to have been stopped by an earth back-fill against the down-stream face, thus preventing extreme temperature variations at the surface of the concrete.

Stony River Dam, West Virginia.—A case with which the speaker has had considerable personal contact is that of the hollow, reinforced concrete dam of the Ambursen type, on Stony River in West Virginia.† This dam is situated in the northern part of the State at an altitude 3 400 ft. above sea level, and is subject to severe freezing weather. It was constructed in 1912-13, failed in part in January, 1914, by the undermining of a too shallow cut-off wall, and was reconstructed in 1914-15, the reconstruction being under the supervision of the speaker.

In the concrete of both the original work and the reconstruction local sandstone was utilized for the coarse aggregate and likewise, by crushing and rolling, for the sand. In both cases the concrete was placed wet, one might say "slushy". The deck slabs were of 1 : 2 : 4 concrete. In no respect had the failure of the dam been due to the quality of the concrete; in fact, the good appearance of the original concrete in the spring of 1914 led to the decision to continue the use of local aggregates. The only excuse for the wet concrete lies in the fact that at that time such was the generally accepted practice.

As part of the reconstruction certain structural modifications were made. That part of the dam which had failed had been of open bulkhead section, but

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1323.

† *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), pp. 907–1100.

this was replaced by means of a second spillway, which, like the original spillway, has a completely enclosing apron on the down-stream side. Furthermore, in order to prevent freezing of the foundation drainage system, installed throughout the entire dam as part of the reconstruction, a concrete housing was built in all the previously open bulkhead bays, except only the bays of lesser height near the ends of the dam.* This housing consists of a curtain-wall of concrete, 12 in. thick, extending for about two-thirds of the height of the dam, together with a roof, 7 in. thick, extending horizontally from the top of the curtain-wall up stream to a junction with the deck.

The water-level fluctuates considerably. There is no fixed regimen which applies one winter after another.

During the first six years of the life of the original concrete, that is, until 1919, practically no evidence of disintegration was discernible. An inspection in the autumn of 1919, however, revealed some spalling of concrete on the up-stream face of one of the original deck slabs near the center of the dam. It is presumed that the spalling originated during the severe winter of 1917-18, for which period a minimum temperature at the dam of 36° below zero Fahr., was reported. At the time of this inspection the spalling extended to a depth of about 5 in., the total thickness of the deck slab at this point being about 14 in. The spalling extended across the entire width of the deck slab (about 13 ft.); this fact, together with the further fact that this slab was the only one which had spalled, indicated that the disintegration may have been due to improper treatment of the concrete in that particular area. In 1921 the disintegrated slab was patched by superimposing a reinforced concrete slab. However, water penetrated between the original and superimposed slabs and the disintegration of the original slab continued, until finally the concrete crumbled and spalled on the lower or down-stream face of the slab in the same region, exposing the reinforcing steel.

In the meantime the two adjacent deck slabs were showing an increasing seepage through the concrete, thus casting doubt on the earlier indication that the slab first showing disintegration had been subject to an isolated instance of faulty workmanship. It is likely, however, that this particular part of the original deck was made of concrete in which, as compared with the remainder of the work, there was aggravated some error in making the concrete, as, for instance, excessive water-cement ratio or improper curing. The slabs in point were undoubtedly cast at a time when there was no freezing and possibly even during hot weather. Further repair work was done during 1924.

The area affected comprises only about one-fourth of 1% of the total deck area of the dam. It is important to note that there is no record of any disintegration below the roof of the housing (which applies to about three-fifths of the bulkhead sections of the dam), and, similarly, that there appears to be no disintegration whatsoever within either the old or the new spillway, and, therefore, completely enclosed, sections of the structure.

The fact that in the new spillway section there has been no disintegration, whether of deck, apron, or buttresses, is the more noteworthy because there has been considerable disintegration in the new spillway mat or channel flooring

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), Plate XII.

and also in the concrete placed during the reconstruction to reinforce the footings in that part of the dam which had not failed. This disintegration of the newer concrete was most marked on horizontal, or only slightly inclined, surfaces. The newer concrete which has suffered disintegration was undoubtedly made with too much water and too little cement, especially considering the quality of the aggregate used, and much of the concrete was placed during the winter. However, as compared with that new concrete which disintegrated, the concrete of the new spillway similarly contained too much water; the cement content for the spillway was greater only in the deck and apron (1 : 2¼ : 3¾), but not in the buttresses, which, like the spillway channel mat, were of 1 : 2¾ : 4¾ mix.

The new deck concrete is not superior to that of the original decks. The former are somewhat thicker at the same elevations, but the speaker believes that it is primarily because of the complete enclosure that the new decks have withstood successfully the severe winter of 1917-18 and appear still to be in excellent condition.

In considering possible causes of the disintegration of the small area of deck slab in the original part of the dam, the question arises whether the difficulty may not have focused at and been due to the ice sheet covering the reservoir in winter. The water surface of the reservoir may actually have been at or slightly above the disintegrated area during the winter of 1917-18; moreover, the fact that the disintegration was first noticeable on the up-stream surface implies that there may have been some plucking action on the part of the ice, thus causing the observed spalling. However, other facts indicate that the ice sheet is not primarily responsible, but rather that it may have accentuated and made obvious a disintegration which had an earlier and more direct cause, namely, percolation of reservoir water through the deck slab and consequent freezing of the slab while in a saturated condition and unprotected on the down-stream side. One fact bearing on this matter is that the water level is by no means fixed seasonably; another is the experience in the case of the Gem Lake Dam where the disintegration extended for many feet below the winter water level of the reservoir.

Limitation of Disintegration to Unenclosed Dams.—The speaker is familiar with or has reports on disintegration of concrete in various other hollow concrete dams, both of the multiple-arch and reinforced concrete deck-slab types. However, persistent inquiry on his part has not revealed to date a single instance where disintegration of concrete has occurred in the decks of hollow concrete dams in which the down-stream faces of the decks are protected against severe freezing temperatures. He would not have been surprised to learn of some cases where obviously poor concrete had disintegrated even under such protected conditions, for, at best, a relatively thin slab subject to percolation from reservoir water is functioning under severe handicaps. The apparently complete absence of disintegration in enclosed parts of hollow concrete dams thus seems all the more remarkable.

Time and space do not warrant consideration of the disintegration of concrete which has taken place in altogether too many solid concrete dams, but it is worthy of note that such disintegration has occurred primarily in regions

where there is severe freezing and in parts of dams which are fully exposed to the prevailing atmospheric conditions. Incidentally, the disintegration which has occurred in such dams appears to be greater as the surfaces approximate the horizontal.

TENTATIVE CONCLUSION

In the light of the foregoing considerations, the speaker has reached the tentative conclusion that hollow concrete dams may be constructed and relied on to be relatively permanent, provided:

1.—The design is good and the proportions of the members liberal, especially as regards thickness of concrete protecting any reinforcing steel.

2.—The concrete is made of proper materials and in a proper manner, in accordance with present knowledge of the art. This involves capable, conscientious, rigid, and authoritative inspection.

3.—The dams are enclosed by means of diaphragms or heat-insulating walls located at or near the down-stream faces of the buttresses.

This tentative conclusion has been reached with full recognition of the fact that hollow concrete structures of necessity can tolerate disintegration to a less extent than solid concrete dams. The point is that disintegration is believed to be avoidable.

The speaker is sufficiently optimistic to believe that hollow concrete dams can be built in freezing climates successfully even without the enclosure. However, in view of the present status of the art of concrete construction and the contingencies which at best enter into any concrete work, he believes it is not conservative to rely entirely on obtaining the proper quality of concrete.

The most important quality to be attained in the deck concrete is, of course, water-tightness and this, in turn, is directly dependent on the density of the concrete itself. Without presuming to have any final knowledge on the point, the speaker doubts the practicability of obtaining in dams reliably water-tight concrete decks by means of surface or membrane coverings. Acknowledging that, in general, such coverings must be of some benefit, he prefers to place his main reliance on properly attained density of the concrete itself.

As to the character of the recommended enclosure, it should be noted that the form of the enclosure in the case of the Stony River Dam was governed by a different consideration. If this point of protecting the entire downstream surface of the decks had been in mind, the enclosure would undoubtedly have been carried to the full height of the dam. A fact of importance developed in the functioning of the Stony River Dam is that within the enclosure the temperature does not fall materially below the freezing point. For instance, during the winter of 1915-16, the lowest temperature within the enclosed part of the dam was + 26° Fahr., whereas the lowest outdoor temperature was — 12° Fahr. Under these conditions no serious ice formation took place within the enclosure. Evidently, the radiation of heat from the reservoir water and from the foundation material into the enclosed space to a large extent offsets the radiation of heat through the enclosing diaphragm from the enclosed space into the outside air.

The down-stream insulating wall may be of concrete, either single or, better still, double; in the latter case, it affords an insulating air space within the wall. Hollow tile also seems appropriate for the purpose. The tile might be coated with stucco or "Gunite" if a more pleasing appearance is desired. Incidentally, such a diaphragm, especially if of concrete, serves as a brace between buttresses.

The water-works reservoir dam, at Coatesville, Pa., of the Ambursen type and constructed under the supervision of Alexander Potter, Assoc. M. Am. Soc. C. E., in 1916-17, is understood to have its non-spilling concrete sections enclosed by an approximately vertical diaphragm of No. 28, four-rib "Hyrib" (a form of metal lathing), covered by a 1-in. thickness of plaster. However, it is further understood that such an enclosure is solely for architectural effect. The speaker also notes with interest that one of the more recent Norwegian dams, the so-called Fjergegn Dam, is enclosed by an insulating wall on the down-stream side.*

MISCELLANEOUS

The speaker will await with much interest the result of the long time test by Nature as to the quality of the concrete used in the Gem Lake reconstruction work, especially in view of the somewhat low cement content. It would be of interest to know in what direction the diagonal cracks extend, that are mentioned by the authors† as having occurred near the up-stream toes of three of the buttresses. It appears not unlikely that these cracks are of the same nature as those mentioned‡ by Mr. Lippincott.

CHARLES W. COMSTOCK,§ M. AM. SOC. C. E.—The dominant note in the preceding discussion seems to be one of condemnation of the concrete used in the original construction. From this opinion the speaker dissents.

Records of concrete tests made during construction are not at hand, but the speaker has had access to them and has studied them carefully. To the best of his recollection compression tests on 6 by 12-in. cylinders at 28 days gave upward of 1 900 lb. per sq. in. Generally this would be regarded as satisfactory.

The speaker made an examination of Gem Lake and Agnew Lake Dams in May, 1924. The exposed portions of both dams, arches and buttresses, were carefully gone over with pick and hammer. Holes were drilled or dug at various places where indications were unsatisfactory. Except for the belt, about 30 ft. wide, of the arches of Gem Lake Dam, which has been described in the paper, the concrete was satisfactory in appearance and perfectly sound, ringing clearly when struck with a hammer. The buttresses at Gem Lake were in perfect condition. There was no spalling or other evidence of deterioration. Agnew Lake Dam was apparently in perfect condition throughout.

The speaker was not connected with the original construction of these dams and has no personal reason for defending the methods or materials used,

* *Proceedings*, Am. Soc. C. E., January, 1926, Papers and Discussions, p. 101.
† *Loc. cit.*, September, 1925, Papers and Discussions, p. 1319.
‡ *Loc. cit.*, January, 1926, Papers and Discussions, pp. 98–99.
§ Engr., Dwight P. Robinson & Co., Inc., New York, N. Y.

but all the evidence indicates that the work done and the results obtained were as good as possible with the materials available.

Mr. Flinn* has suggested that concrete materials should be selected with greater care than is usually bestowed upon them, that they should be carefully analyzed and scientifically proportioned. While the soundness of this suggestion will not be questioned, and it is certain that the best results cannot otherwise be obtained, it should be remembered that the constructor is nearly always limited by considerations of cost to the sand and coarse aggregate afforded by the surrounding territory. Cement is purchased under standard specifications with which all established brands comply. Defective concrete is rarely due to poor cement, although sometimes to improper manipulation.

In the speaker's opinion unsatisfactory concrete results more often from poor sand than from any other one cause, but in this material the constructor frequently has little or no choice. The alternative is to use the best that is available and be content with concrete of quality inferior to the ideal.

At the time of the reinforcement of Gem Lake Dam the sandpits used in the original construction were entirely submerged and inaccessible. The nearest natural sand would have had to be hauled 16 miles and then drawn up the inclines and barged across Agnew Lake, as was done with cement and lumber. The cost was prohibitive. It was necessary to grind the country rock to obtain sand. Whether the new concrete will be superior to the old remains to be seen.

A word as to Agnew Lake Dam—it is about 550 ft. lower than Gem Lake and was built to afford storage for one or two seasons prior to the completion of Gem Lake Dam. Since the completion of the upper dam water stands almost constantly at spillway level in Agnew Lake, thus protecting the water face against freezing on that side. Agnew Lake Dam is only a little more than 30 ft. high, and snowdrifts pile up to its full height on the lower side every winter. This affords ample protection against the freezing which has damaged a part of the Gem Lake structure, and accounts for the present sound condition of the lower dam.

After a careful study of both these dams the speaker was unable to find any other explanation of the peculiar nature and location of the damage than that given in the paper. He does not believe that the facts square with any assumption of poor concrete in the original construction. Whether it would have been better to have chosen an entirely different type of dam in the first place is another question.

GEORGE W. HOWSON,† ASSOC. M. AM. SOC. C. E. (by letter).‡—This very frank discussion of the failure of the concrete in the arches of the Gem Lake Dam is extremely interesting to those engaged in the design and construction of dams, and carries a warning to those building thin exposed walls subject to water pressure. Many theories may be advanced as to the cause of the failure, but all are without conclusive proof. The most logical conclusions have been reached by the authors that moisture found its way into

* *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 2015.

† Res. Engr., Dix River Dam Project, Burgin, Ky.

‡ Received by the Secretary, January 29, 1926.

the concrete and there froze, causing disintegration. They rightfully conclude that field construction conditions make extremely difficult, if not impossible, a concrete absolutely impervious to the slightest penetration of moisture. It is gratifying to note that they conclude that the rock-fill type has many advantages over other types of dams in such a locality as Gem Lake. The writer believes that the virtues of the rock-fill dam extend further than to localities such as Gem Lake. It adapts itself to certain foundations, such as loose rock or cemented gravel in stream beds, requiring a minimum of excavation compared with masonry dams. At present, there is a tendency toward the popularity of the rock-fill dam, probably due to the increasing familiarity of engineers with it. The writer has knowledge of five large proposed dams, all of the rock-fill type.

The rock-fill dam is not susceptible to such a refined degree of analysis as the multiple-arch, and owing to the very nature of the structure it is not necessary that it should be. The design is largely a matter of experience and not of refined computation.

Recently, the writer has been in charge of the construction of the super rock-fill dam on the Dix River in Kentucky, designed by L. F. Harza, M. Am. Soc. C. E. This dam is 275 ft. high above the stream bed and 296 ft. above the foundation excavations. It contains 1 800 000 cu. yd. of material and is by far the master effort yet attempted in this type of construction.

When properly designed the rock-fill dam has, for a water-tight skin spread over the exposed up-stream face, a layer of reinforced concrete which, in the larger dams, generally varies from a maximum of 2 ft. to a minimum of 6 in. in thickness. In this respect the rock-fill type has something in common with the multiple-arch; both rely on a thin slab of concrete for water-tightness. In case of the rock-fill dam the concrete slab is protected from the elements on the lower side by the mass of the rock-fill over which it is spread and on which it relies for support. In the multiple-arch dam the thin arches are exposed on the lower side to the elements, and it is believed to be this exposure at Gem Lake that caused the failure.

The large rock-fill dam must have a concrete water-skin of a quality equal to, if not better than, the arches of a concrete structure. It must be as nearly impervious as possible and, in addition, be able to develop great strength to meet indeterminate stresses due to movements in the rock-fill itself. By proper construction of the body of the dam, movements of the fill may be partly controlled and reduced in amount, but nevertheless they are present in a greater or less degree, depending on the intelligence, guided by experience, used in the supervision of the construction of the fill. Once the apron of concrete is poured against the fill it must conform to the movements or take stresses sufficient to overcome them.

No concrete apron spread over the face of a large rock-fill dam would be able to resist major movements of the dam, nor is it ever intended that it should in a well-designed apron; but the apron may be so designed and so constructed that it can overcome tendencies to small local movements. This is a matter of the fine combination of proper design and good supervision so necessary in dam construction.

THE IMPROVED VENTURI FLUME

Discussion*

By Carl Rohwer, Assoc. M. Am. Soc. C. E.

Carl Rohwer,† Assoc. M. Am. Soc. C. E. (by letter).‡—Mr. Parshall lays particular stress on the effect of submergence. According to his report, the degree of submergence is immaterial for amounts of less than 70 per cent. Compared with the weir this is unusual, as shown by Table 8,§ which is a comparison between the effect of submergence on a 4-ft. rectangular weir and a 4-ft. improved Venturi flume. Table 8 gives for different percentages of submergence the ratio of the submerged flow discharge to the free flow discharge.

TABLE 8.—A Comparison of the Effect of Submergence on a 4-Foot Rectangular Weir and a 4-Foot Improved Venturi Flume.

4-Foot Rectangular Weir

Test No.	Upper head, in feet.	Lower head, in feet.	Submergence ratio.	Discharge, in second-feet.	Free flow discharge, in second-feet.	Discharge ratio.
1858	0.400	0.300	0.750	2.155	3.316	0.650
1860	0.400	0.200	0.500	2.761	3.316	0.833
1863	0.400	0.100	0.250	3.128	3.316	0.943
1868	0.400	0.000	0.000	3.333	3.316	1.005
1913	0.600	0.500	0.833	3.351	6.000	0.558
1877	0.600	0.400	0.667	4.406	6.000	0.734
1833	0.600	0.300	0.500	5.046	6.000	0.841
1856	0.600	0.200	0.333	5.502	6.000	0.917
1881	0.600	0.100	0.167	5.853	6.000	0.976
1891	0.800	0.700	0.875	4.686	9.161	0.512
1893	0.800	0.600	0.750	6.167	9.161	0.673
1894	0.800	0.500	0.625	7.069	9.161	0.772
1904	0.800	0.400	0.500	7.790	9.161	0.850
1929	0.800	0.300	0.375	8.341	9.161	0.911
1898	0.800	0.200	0.250	8.885	9.161	0.970
1852	1.000	0.900	0.900	5.952	12.705	0.468
1849	1.000	0.800	0.800	8.016	12.705	0.631
1835	1.000	0.700	0.700	9.177	12.705	0.722
1839	1.000	0.600	0.600	9.550	12.705	0.752
1854	1.000	0.500	0.500	10.913	12.705	0.859
1840	1.000	0.400	0.400	11.584	12.705	0.908
1862	1.000	0.300	0.300	12.041	12.705	0.948
1922	1.300	1.200	0.923	7.936	18.71	0.424
1924	1.300	1.100	0.846	10.514	18.71	0.562
1927	1.300	1.000	0.770	12.301	18.71	0.657
1917	1.300	0.900	0.682	13.799	18.71	0.738
1931	1.300	0.800	0.615	14.702	18.71	0.786
1925	1.300	0.700	0.539	15.546	18.71	0.831

* Discussion of the paper by Ralph L. Parshall, Affiliate, Am. Soc. C. E., continued from February, 1926, *Proceedings*.

† Assoc. Irrig. Engr., Div. of Agri. Eng., U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

‡ Received by the Secretary, December 4, 1925.

§ Prepared from data of the Office of Irrigation Investigations, Div. of Agricultural Eng., Bureau of Public Roads, U. S. Dept. of Agriculture, co-operating with the Colorado Agricultural Experiment Station.

TABLE 8.—(*Continued.*)

4-Foot Improved Venturi Flume.

6405	0.598	0.333	0.557	6.98	7.11	0.982
6406	0.792	0.761	0.961	7.02	11.07	0.634
6407	1.155	1.146	0.992	7.02	20.08	0.350
6398	0.966	0.599	0.620	14.94	15.15	0.986
6399	1.014	0.791	0.780	15.13	16.35	0.925
6400	1.248	1.180	0.946	15.43	22.69	0.680
6401	1.735	1.708	0.984	15.76	38.17	0.413
6395	1.339	0.977	0.730	24.55	25.36	0.968
6390	1.340	0.828	0.618	25.60	25.39	1.008
6394	1.421	1.177	0.828	24.55	27.85	0.882
6393	1.624	1.508	0.929	24.40	34.40	0.709
6392	2.079	2.020	0.972	24.98	50.78	0.492
6372	1.644	1.070	0.651	35.66	35.05	1.017
6373	1.702	1.295	0.761	35.40	37.03	0.956
6374	1.804	1.519	0.842	35.16	40.59	0.866
6375	2.088	1.955	0.936	33.72	51.13	0.659
6376	2.441	2.384	0.977	29.54	65.42	0.452
6381	1.973	1.286	0.652	47.00	46.75	1.005
6380	1.994	1.300	0.652	47.80	47.55	1.005
6379	2.001	1.309	0.654	47.92	47.81	1.002
6382	2.008	1.475	0.735	46.32	48.07	0.964
6383	2.082	1.688	0.811	45.67	50.90	0.897
6384	2.280	2.064	0.905	43.42	58.74	0.739

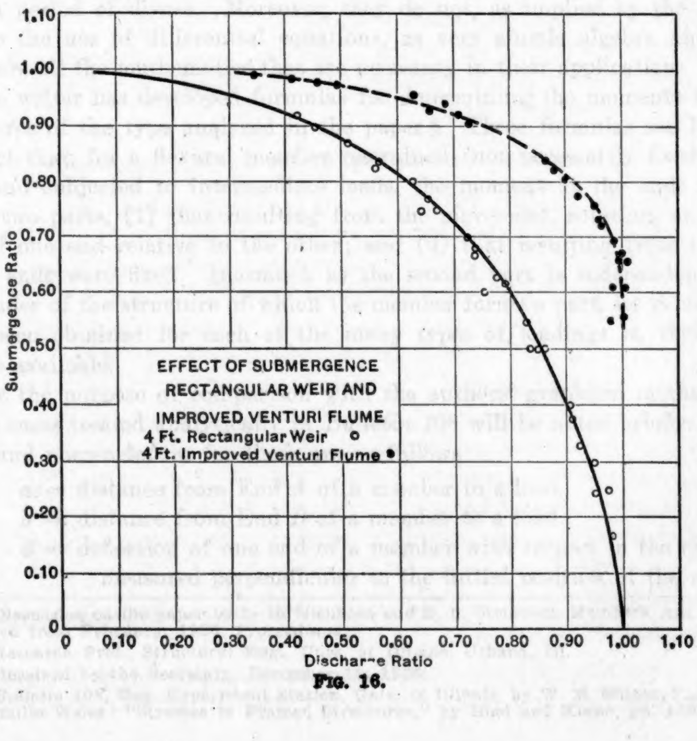


Fig. 16.

The percentages of submergence for both the weir and the Venturi flume are based on heads referred to the crest level. On account of the nature of the data it was not possible to obtain similar percentages of submergence for both the weir and the Venturi flume, and in order to make direct comparisons possible, Fig. 16 was prepared.

A study of the diagram shows that for submergences below 70%, there is little effect on the discharge of the Venturi flume, while the discharge from the weir is materially reduced. For submergences from 70 to 90%, the discharge ratios for both the weir and the Venturi flume decrease at about the same rate, but for values greater than 90% the effect is greater on the Venturi flume.

The fact that submergence does not affect the flow of the improved Venturi flume for ratios less than approximately 70% is to be expected. It has been shown* that at the control point, which in this case is at the crest, the depth cannot drop below two-thirds of the total head at the upper gauge point. In other words, submergence does not affect the discharge until the down-stream depth is sufficient to cause the depth at the control point to exceed two-thirds of the total head at the upper gauge point. For the improved Venturi flume, this apparently occurs when the down-stream head, as measured by Mr. Parshall, exceeds 70% of the depth at the upper gauge.

* "The Hydraulic Jump, in Open Channel Flow at High Velocity," by Karl R. Kennison, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 346. "Flow of Water Through Contractions," by E. W. Lane, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1168.

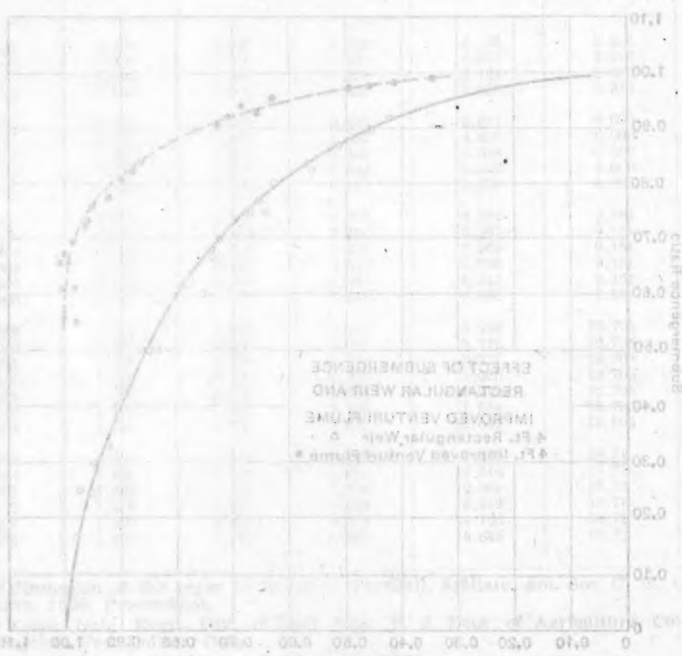


Fig. 16.

MOMENTS IN RESTRAINED AND CONTINUOUS BEAMS BY THE METHOD OF CONJUGATE POINTS

Discussion*

By Messrs. W. M. Wilson, S. M. Cotten, and A. T. Granger.

W. M. Wilson,† M. Am. Soc. C. E. (by letter).‡—For analyzing statically indeterminate flexural members the authors have presented some interesting graphical methods which they claim are much simpler than those previously available. In considering the relative merits of two methods the writer has observed that one's own "child" is invariably a favorite; also that a graphical method appeals to one type of mind whereas an algebraic method appeals to other types. Moreover, a method—even one that is rapid when completely mastered—only results in delay if the successive steps must be reviewed each time it is applied. The writer believes that the algebraic methods available are as short as the graphical method presented by the authors, that they have as great a range of application, and that they can be reviewed more quickly after a period of disuse. Moreover, they do not, as implied by the authors, involve the use of differential equations, as very simple algebra and arithmetic are all the mathematics that are necessary in their application.

The writer has developed formulas for determining the moments in many structures of the type analyzed in the paper.§ These formulas are based on the fact that, for a flexural member restrained (not necessarily fixed) at the ends and subjected to intermediate loads, the moment at the ends is made up of two parts, (1) that resulting from the movement, rotation, or translation of one end relative to the other; and (2) that resulting from the loads if the ends were fixed. Inasmuch as the second part is independent of the remainder of the structure of which the member forms a part, its value having once been obtained for each of the many types of loadings is, thenceforth, always available.

For the purpose of comparison with the authors' graphical methods some of the cases treated analytically in *Bulletin 108* will be noted briefly. Definitions and nomenclature for these are as follows:

a = distance from End A of a member to a load.

b = distance from End B of a member to a load.

d = deflection of one end of a member with respect to the other end, measured perpendicular to the initial position of the member.

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* Discussion on the paper by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., continued from February, 1926, *Proceedings*.

† Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

‡ Received by the Secretary, December 11, 1925.

§ *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, by W. M. Wilson, F. E. Richart, and Camillo Weiss; "Stresses in Framed Structures," by Hool and Kinne, pp. 485 to 614.

e = eccentricity of the load.

h = vertical height of a structure.

k = error in the resisting moment due to neglect of the shearing strain.

l = length of a member.

m = change in the rate of loading in a unit distance.

n = ratio of K of the top member to K of the left-hand column for a four-sided frame.

p = ratio of K of the top member to K of the bottom member for a four-sided frame.

s = ratio of K of the top member to K of the right-hand column for a four-sided frame.

u = load per unit of length (variable).

w = uniformly distributed load per unit of length.

A = area of section of a member.

C_{AB} = resisting moment at End A of a member, AB, fixed at both ends and having both ends at the same level.

E = modulus of elasticity in tension and compression.

F = area of the moment diagram of a simple beam.

H_{AB} = resisting moment at End A of a member, AB, fixed at A and hinged at B and having both ends at the same level.

I = moment of inertia of section of a member.

K = ratio of moment of inertia of section to length of a member.

M = moment of an external couple.

M_{AB} = resisting moment acting at the end, A, of a member, AB.

M_{BA} = resisting moment acting at the end, B, of a member, AB.

P = concentrated load.

$R = \frac{d}{l}$ = ratio of the deflection of one end of a member (with respect to the other end) to the length of the member.

W = total distributed load on a member.

$\alpha = n^2 + 2pn + 2n + 3p$, for a symmetrical four-sided frame.

$\beta = 6n + p + 1$, for a symmetrical four-sided frame.

$\Delta = 22\,(p\,n\,s + p\,s + n\,s + n\,p) + 2\,(p^2\,s + p\,s^2 + p\,n^2 + p^2\,n + s^2 + s + n^2 + n) + 6\,(n^2\,s + n\,s^2 + p^2 + p)$, for a rectangular frame.

$\Delta_0 = 2\,[n\,s\,(4 + 3\,q + 4\,q^2) + (s^2 + s) + q^2\,(n^2 + n) + 3\,(q^2\,s\,n^2 + s^2 n)]$ for a two-legged rectangular bent with unequal legs.

$\Delta_0 = 2\,(3ns^2 + 11\,ns + s^2 + s + 3n^2s + n^2 + n)$, for a two-legged rectangular bent.

θ = change in the slope of the tangent to the elastic curve of a member.

The signs of the quantities used in the equations are determined by the following rules:

(1) θ is positive (+) when the tangent to the elastic curve is turned in a clockwise direction.

(2) R is positive (+) when the member is deflected in a clockwise direction.

(3) The moment of the internal stresses on a section is positive (+) when the internal couple acts in a clockwise direction upon the part of the member considered.

(4) If the moment of the external forces on the member about the end at which the moment is to be determined is positive (+), the sign before the constant is minus (—); if the moment of the external forces about the end at which the moment is to be determined is negative (—), the sign before the constant is (+). With the external forces acting downward, (Fig. 46) for the moment at A, C_{AB} and H_{AB} are preceded by a minus (—) sign, but for the moment at B, C_{BA} and H_{BA} are preceded by a plus (+) sign.

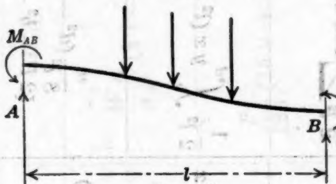


FIG. 46.

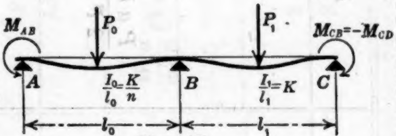


FIG. 47.

The moment at the ends of a member, AB (Fig. 46), is given by Equations (30) to (33):

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) \mp C_{AB} \quad \ldots\ldots\ldots\ldots (30)$$

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) \pm C_{BA} \quad \ldots\ldots\ldots\ldots (31)$$

If End B is hinged,

$$M_{AB} = E\,K\,(3\,\theta_A - 3\,R) \mp H_{AB} \quad \ldots\ldots\ldots\ldots (32)$$

If End A is hinged,

$$M_{BA} = E\,K\,(3\,\theta_B - 3\,R) \pm H_{BA} \quad \ldots\ldots\ldots\ldots (33)$$

The symbol, C, represents the moment that would be produced by the loads if the ends of the member were fixed, C_{AB} being the moment at End A and C_{BA} being the moment at End B. It is sometimes desirable for algebraic convenience, to introduce the moment produced by the given loads at the fixed end of a beam that is fixed at one end and hinged at the other. This moment has been represented by H.

The moment at the end of any flexural member, AB, can be represented by some one of these equations. Values of C and H for various loads are given in Tables 3 and 4, Table 3 being for unsymmetrical systems of loads and Table 4 being for loads symmetrical about the center of the member.

The analyses of indeterminate structures consist of the derivation of equations for the moments in statically indeterminate flexural members by the application of Equations (30) to (33) and the substitution of the values of C and H. It is unnecessary to reproduce the derivation of these equations.

The Equation of Three Moments for the continuous girder of Fig. 47 is:

$$n\,M_{AB} + 2\,M_{BC}\,(n + 1) + M_{CD} = -2\,(n\,H_{BA} + H_{BC}) \ldots\ldots (34)$$

Equation (34) is applicable to any two adjacent intermediate spans of any continuous girder having supports on the same level, no matter what the type

TABLE 3.—VALUES OF CONSTANTS, C AND H, FOR DIFFERENT SYSTEMS OF LOADS.

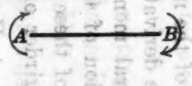
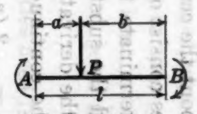
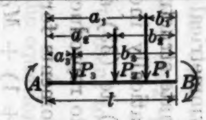
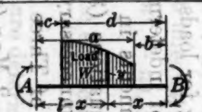
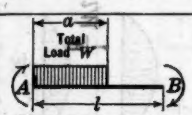
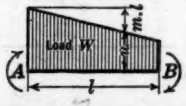
No.	Condition of loading.	C_{AB}.	C_{BA}.	H_{AB}.	H_{BA}.
1	 No intermediate external loads	0	0	0	0
2	 Single concentrated load at any point	$\frac{P\,a\,b^2}{l^2}$	$\frac{P\,b\,a^2}{l^2}$	$\frac{P\,a\,b}{2\,l^2}(l+b)$	$\frac{P\,a\,b}{2\,l^2}(l+a)$
3	 Any number of concentrated loads	$\frac{1}{l^2}\sum P\,a\,b^2$	$\frac{1}{l^2}\sum P\,b\,a^2$	$\frac{1}{2\,l^2}\sum P\,a\,b\,(l+b)$	$\frac{1}{2\,l^2}\sum P\,a\,b\,(l+a)$
4	 Any distributed load over any portion at the member	$\frac{1}{l^2}\int_b^d y\,x^2\,(l-x)\,d\,x$	$\frac{1}{l^2}\int_b^d y\,x\,(l-x)^2\,d\,x$	$\frac{1}{2\,l^2}\int_b^d y\,x\,(l^2-x^2)\,d\,x$	$\frac{1}{2\,l^2}\int_b^d y\,x\,(l-x)$ $(2\,l-x)\,d\,x$
5	 Uniform load over any portion of the member	$\frac{w}{12\,l^2}\Big[d^3\,(4\,l-3d)$ $-\,b^3\,(4\,l-3\,b)\Big]$	$\frac{w}{12\,l^2}\Big[a^3\,(4\,l-3\,a)$ $-\,c^3\,(4\,l-3\,c)\Big]$	$\frac{w}{8\,l^2}(d^2-b^2)$ $(2\,l^2-b^2-d^2)$	$\frac{w}{8\,l^2}(a^2-c^2)$ $(2\,l^2-a^2-c^2)$

TABLE 3.—(*Continued.*)

No.	Condition of loading.	C_{AB}.	C_{BA}.	H_{AB}.	H_{BA}.
6	 Uniform load over portion of member at one end	$\frac{W a}{12 l^2}(3 a^2 - 8 a l + 6 l^2)$	$\frac{W a^2}{12 l^2}(4 l - 3 a)$	$\frac{W a}{8 l^2}(2 l - a)^2$	$\frac{W a}{8 l^2}(2 l^2 - a^2)$
7	 Uniformly varying load over entire member	$\frac{l^2}{60}(5 u + 3 m l)$	$\frac{l^2}{60}(5 u + 2 m l)$	$\frac{l^2}{120}(15 u + 8 m l)$	$\frac{l^2}{120}(15 u + 7 m l)$
8	 Load varying uniformly from zero at one end to a maximum at the other end.	$\frac{W l}{10}$	$\frac{W l}{15}$	$\frac{2}{15} W l$	$\frac{7}{60} W l$
9	 Load varying uniformly from zero at any point to a maximum at one end.	$\frac{W a}{30 l^2}(3 a^2 - 10 a l + 10 l^2)$	$\frac{W a^2}{30 l^2}(5 l - 3 a)$	$\frac{W a}{60 l^2}(3 a^2 - 15 a l + 20 l^2)$	$\frac{W a}{60 l^2}(10 l^2 - 3 a^2)$
10	Any load symmetrical about Center Line.  Any load symmetrical about center of member.	$\frac{F}{l}$*	$\frac{F}{l}$*	$\frac{3}{2} \cdot \frac{F}{l}$*	$\frac{3}{2} \cdot \frac{F}{l}$*

* Values of $\frac{F}{l}$ for different loads are given in Table 4.

TABLE 4.—VALUES OF C AND H FOR LOADS SYMMETRICAL ABOUT THE CENTER OF THE MEMBER.

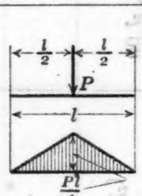
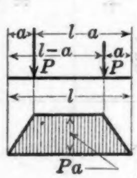
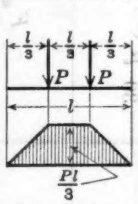
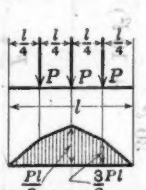
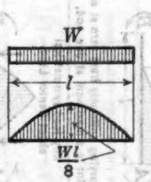
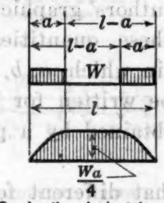
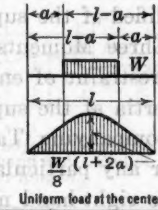
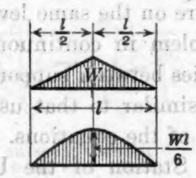
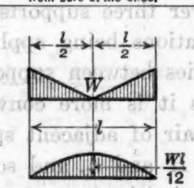
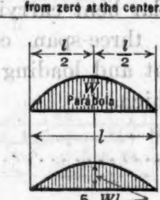
No.	Loading and Moment Diagram	$C_{AB} = C_{BA} = \frac{F}{l}$	$H_{AB} = H_{BA} = \frac{3}{2}\frac{F}{l}$
1	 Single load at the center.	$\frac{1}{8}Pl$	$\frac{3}{16}Pl$
2	 Two equal loads.	$\frac{Pa}{l}(l-a)$	$\frac{3}{2}\frac{Pa}{l}(l-a)$
3	 Equal loads at the third points.	$\frac{2}{9}Pl$	$\frac{1}{3}Pl$
4	 Equal loads at the quarter-points.	$\frac{5}{16}Pl$	$\frac{15}{32}Pl$
5	 Uniform load over entire span.	$\frac{1}{12}Wl$	$\frac{1}{8}Wl$

TABLE 4.—(*Continued.*)

No.	Loading and Moment Diagram	$C_{AB} = C_{BA} = \frac{F}{l}$	$H_{AB} = H_{BA} = \frac{3}{2}\,\frac{F}{l}$
6	 Equal uniform loads at the ends.	$\frac{Wa}{12l}(3l-2a)$	$\frac{Wa}{8l}(3l-2a)$
7	 Uniform load at the center.	$\frac{W}{12l}(l^2+2al-2a^2)$	$\frac{W}{8l}(l^2+2al-2a^2)$
8	 Load increasing uniformly from zero at the ends.	$\frac{5}{48}Wl$	$\frac{5}{32}Wl$
9	 Load increasing uniformly from zero at the center.	$\frac{1}{16}Wl$	$\frac{3}{32}Wl$
10	 Load varying as the ordinates of a parabola	$\frac{1}{10}Wl$	$\frac{3}{20}Wl$

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1606.

of loading, length of span, or value of I, so long as I does not vary between the supports. The symbol, n, represents $\frac{I}{l}$ for the right-hand span divided by $\frac{I}{l}$ for the left-hand span. It should be noted that quantities corresponding to n and H must be computed in connection with the authors' graphical method before the diagrams can be constructed. With these quantities known, Equation (34) takes the form, $ax + by + cz = K$, in which, a, b, c, and K are numerical quantities. A similar equation can be written for each pair of adjacent spans and the values of the moments obtained by a process of elimination.

The Equation of Three Moments takes a somewhat different form when applied to two adjacent spans at one end of a girder, depending on the condition of restraint at the ends; it is still further modified if the supports are not on the same level. Forms of the Equation of Three Moments covering all possible combinations of loading, length of span, restraint of ends, settlement of supports, and changes in the moments of inertia at the supports are contained in Table 5. The *P's* in the diagrams accompanying Table 5 are symbolic of any system of loading, the value of H for any particular loading being given in Tables 3 and 4. The first term of the right-hand member of these equations is zero in all cases if the supports are on the same level. These equations make possible the solution of any problem in continuous girders except those involving members the I of which varies between supports. These latter problems could also be solved in a manner similar to that used by the authors, substituting algebraic for graphic solution of the equations.

Bulletin 108 of the Engineering Experiment Station of the University of Illinois, previously mentioned, contains equations giving directly the moment at each support of girders continuous over three supports and also for those continuous over four supports, the equations being applicable for all possible cases except that in which the I varies between supports. For girders continuous over more than four supports it is more convenient to write the Equation of Three Moments for each pair of adjacent spans, substituting the numerical values for the constants, H and n, and solving the resulting numerical equations by a process of elimination.

The authors have treated the two-legged rectangular bent carrying a vertical load on the top by substituting an equivalent three-span continuous girder. This method can be used only when the bent and loading are symmetrical about a vertical center line (Fig. 48), for which,

$$M_{AD} = -M_{BC} = \frac{3\,C_{AB}}{2\,n + 3} \quad \text{..................} (35)$$

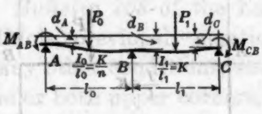
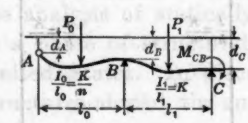
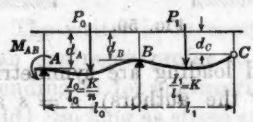
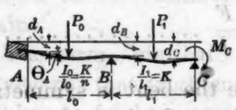
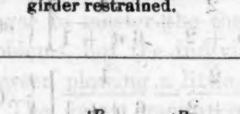
Certainly this solution is shorter than the graphical construction given by the authors (Fig. 16*), especially since quantities corresponding to C_{AB} and n must be determined before the graphical construction is begun. The algebraic

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1606.

TABLE 5.—Continuous Girders—Equations of Three Moments.

(Supports on Different Levels;* Any System of Vertical Loads.†

$K = \frac{I}{l}$ for right-hand span; $\frac{K}{n} = \frac{I}{l}$ for left-hand span.)

No.	Portion of girder considered.	Equations of three moments.
(a)	 Intermediate spans.	$n M_{AB} + 2 M_{BC} (n+1) + M_{CD} = \frac{6 E K}{l_0 1} [l_1 (d_B - d_A) - l_0 (d_C - d_B)] - [2 H_{BC} + n H_{BA}]$
(b)	 Two adjacent spans at left-hand end. End of girder hinged.	$2 M_{BC} (n+1) + M_{CD} = \frac{6 E K}{l_0 l_1} [l_1 (d_B - d_A) - l_0 (d_C - d_B)] - 2 [n H_{BA} + H_{BC}]$
(c)	 Two adjacent spans at right-hand end. End of girder hinged.	$n M_{AB} + 2 M_{BC} (n+1) = \frac{6 E K}{l_0 l_1} [l_1 (d_B - d_A) - l_0 (d_C - d_B)] - 2 [n H_{BA} + H_{BC}]$
(d)	 Two adjacent spans at left-hand end. End of girder restrained.	If M_{AB} is known: $2 M_{BC} (n+1) + M_{CD} = \frac{6 E K}{l_0 l_1} [l_1 (d_B - d_A) - l_0 (d_C - d_B)] - 2 [H_{BC} + n H_{BA}] - n M_{AB}$ If θ_A is known: $M_{BC} (4 + 3n) + 2 M_{CD} = \frac{6 E K}{l_0 l_1} [3 l_1 (d_B - d_A) - 2 l_0 (d_C - d_B)] + 2 n H_{AB} - 4 [H_{BC} + n H_{BA}] - 6 E K \theta_A$ If the girder is fixed at A, $\theta_A = 0$
(e)	 Two adjacent spans at right-hand end. End of girder restrained.	If M_{CB} is known: $n M_{AB} + 2 M_{BC} (n+1) = \frac{6 E K}{l_0 l_1} [l_1 (d_B - d_A) - l_0 (d_C - d_B)] - 2 [H_{BC} + n H_{BA}] + M_{CB}$ If θ_C is known: $2 n M_{AB} + M_{BC} (4n + 3) = \frac{6 E K}{l_0 l_1} [2 l_1 (d_B - d_A) - 3 l_0 (d_C - d_B)] - 4 [H_{BC} + n H_{BA}] + 2 H_{CB} + 6 E K \theta_C$ If the girder is fixed at C, $\theta_C = 0$

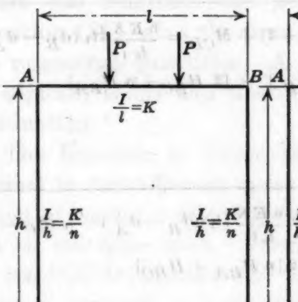
* If there is no settlement of supports, let all values of d in these equations equal zero.

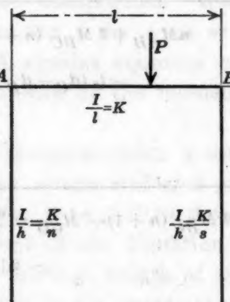
† If there are no loads on girder except at supports, let all values of H in these equations equal zero.

method has the additional advantage that it can be applied when both the bent and loading are unsymmetrical (Fig. 49), for which,

$$M_{BC} = -\frac{3}{2}\left[\frac{C_{AB} + C_{BA}}{n + s + 3}\right] \ldots\ldots\ldots\ldots\ldots (36)$$

$$M_{AD} = \frac{3}{2}\left[\frac{C_{AB} + C_{BA}}{n + s + 3}\right] = -M_{BC} \ldots\ldots\ldots\ldots (37)$$





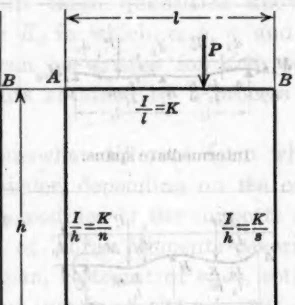


FIG. 48. FIG. 49. FIG. 50.

If the column bases are fixed and the bent and loading are symmetrical about a vertical center line (the case treated by the authors), $n = s$ and $C_{AB} = C_{BA} = \frac{F}{l}$. The equations for the moments then become:

$$M_{AB} = M_{BC} = -\frac{2}{(n+2)}\,\frac{F}{l} \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

$$M_{CB} = -M_{DA} = -\frac{1}{n+2}\,\frac{F}{l} \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

For the more general case with fixed bases where the bent is symmetrical but the loading is not (Fig. 50), a case for which the author's method is not applicable,

$$M_{AB} = -\frac{1}{2}\left\{C_{BA}\left[\frac{2}{n+2} - \frac{1}{6n+1}\right] + C_{AB}\left[\frac{2}{n+2} + \frac{1}{6n+1}\right]\right\}.(40)$$

$$M_{BC} = -\frac{1}{2}\left\{C_{BA}\left[\frac{2}{n+2} + \frac{1}{6n+1}\right] + C_{AB}\left[\frac{2}{n+2} - \frac{1}{6n+1}\right]\right\}.(41)$$

$$M_{CB} = -\frac{1}{2}\left\{C_{BA}\left[\frac{1}{n+2} - \frac{1}{6n+1}\right] + C_{AB}\left[\frac{1}{n+2} + \frac{1}{6n+1}\right]\right\}.(42)$$

$$M_{DA} = \frac{1}{2}\left\{C_{BA}\left[\frac{1}{n+2} + \frac{1}{6n+1}\right] + C_{AB}\left[\frac{1}{n+2} - \frac{1}{6n+1}\right]\right\}.(43)$$

For the still more general case in which neither the bent nor the loading are symmetrical,

$$M_{AB} = -\frac{1}{\Delta_0}\left\{C_{BA}(10\,n\,s + s^2) + C_{AB}(11\,n\,s + 2\,s^2 + 2\,s + 2\,n)\right\}\ldots(44)$$

$$M_{BC} = -\frac{1}{\Delta_0}\left\{C_{BA}(11\,n\,s + 2\,n^2 + 2\,n + 2\,s) + C_{AB}(10\,n\,s + n^2)\right\}\ldots(45)$$

$$M_{CB} = -\frac{1}{\Delta_0}\left\{C_{BA}(7\,n\,s - 2\,n^2 - 2\,n + s) + C_{AB}(8\,n\,s - n^2 + 3\,n)\right\}\ldots(46)$$

$$M_{DA} = \frac{1}{\Delta_0}\left\{C_{BA}(8\,n\,s - s^2 + 3\,s) + C_{AB}(7\,n\,s - 2s^2 - 2\,s + n)\right\}\ldots(47)$$

Bulletin 108 of the Engineering Experiment Station of the University of Illinois, previously mentioned, also contains the treatment of the bent under many other conditions, such as horizontal loads on one or both legs, couples at one or both upper corners, and settlement, spread, and rotation of foundations. It also contains analyses of many similar structures not treated in the paper.

The writer welcomes the authors' contribution to American literature on the analysis of statically indeterminate structures because it gives publicity to a phase of structural engineering that too long has been slighted in the United States. In discussing the use of economic, statically indeterminate structures abroad, the authors state*:

"* * * but the development of the same class of structures in the United States appears to have been retarded by a prevailing impression that the required analysis is involved and tedious. The deterring influence of such an impression is generally under-estimated."

The writer is in complete accord with this statement. Too many American engineers prefer to spend $10 for construction work in order to avoid spending $1 for engineering work. And they wonder why engineers are poorly paid! So long as stress analysis is limited to the selection of data from a handbook, it will be on a plane with clerical work and the computer will receive, properly, a clerk's pay. The Medical Profession has pretty well rid itself of the quack with his pills for all diseases and it is time that the engineer rids himself of the "handbook artist" with coefficients for all conditions of restraint.

The term, "statically indeterminate," has been too long the "ghost in the graveyard" that frightens the student of structural engineering, but it is really nothing to frighten any one willing to do real "honest-to-goodness" studying. From statics to the analysis of statically indeterminate structures is no great step to the student who thoroughly understands statics. It probably takes longer to master the theory and it certainly takes longer to work numerical problems, but the individual steps are no greater; it is simply the difference between plowing a little field and plowing a big one.

The great amount of literature that is now being published, the large amount of time that the engineering schools are giving to the subject, and the extent to which practicing designers are using statically indeterminate structures and are computing secondary stresses, are evidence that the American engineer is becoming more and more convinced of the value of a careful analysis of structures.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1592.

S. M. Cotten,* Assoc. M. Am. Soc. C. E. (by letter).†—In the sense that the authors have presented, in the technical language of the present day and as a comprehensive whole, the essential features of methods of analysis which heretofore have been available only in rather fragmentary and not readily accessible form, they have performed a great service to the bulk of the profession, and one that will undoubtedly receive its due appreciation.

There is, however, nothing essentially new in the final applied methods of analysis. Some old things are treated to new names, some established things are used without name or explanation, and the true major premise is proved as a secondary consequence. In the writer's opinion, these substitutions and re-arrangements have been to the detriment of both theory and presentation, and it would have been better for the authors to have accepted principles heretofore developed that were available for their purposes. The ultimate result would have been the same, and the development of it much more direct and tangible.

Relation of Authors' to Previous Methods.—The most peculiar feature of the paper is that the authors do not discuss the points, U and V, except to state that they are at a certain point on the span and that their height is given by a certain formula. The only theory or explanation given is that they represent the "influence of the applied loading", which is a somewhat indefinite statement in view of the fact that they appear to be the really essential element in the whole plan. Considerable space is devoted to the proof for the location and value of the point, T_{1-2}, all of which is predicated on the properties of the points, U and V. The "pennant diagram", in turn, has the point, T_{1-2}, as an essential element. Again, in the proof of the construction for the "conjugate points", the properties assigned to the U and V-points are essential to the demonstration, and it might be said that this is really a proof that the properties assigned to the points are correct. Because of the evident importance of these points, and the fact that they must possess certain definite properties in order to satisfy the authors' whole theory, the explanation given for them is decidedly inadequate. It would be interesting to know what suggested these points and their peculiar properties to the authors, and how their function may be defined, as related to the other parts of the authors' theory.

Viewed from the standpoint of methods previously developed and in use, the U and V-points are easily explained and their significance is known, for they are nothing other than Fidler's "characteristic points". These points were an original discovery of Fidler and the basis of his method of analysis, which in the writer's opinion is one of the really great contributions to the technique of engineering design.‡ In view of the publicity and the more or less current use of the method, it is quite remarkable that the authors should have devel-

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† Received by the Secretary, December 14, 1925.

‡ Detailed descriptions and proofs of the method were published in *Minutes of Proceedings*, Inst. C. E., Vol. 74, p. 196, 1883, and in a "Practical Treatise on Bridge Construction," by T. Claxton Fidler. The method has since been treated by various writers, and was abstracted in a paper entitled "Graphical and Mechanical Analysis of Frames," by F. E. Richart, Assoc. M. Am. Soc. C. E., and W. M. Wilson, M. Am. Soc. C. E., *Engineering and Contracting*, June 23, 1920.

oped the identical points in their method, without ever suspecting that their discovery had been antedated by about forty years, and that instead of the mere *U* and *V* designations their rightful name is Fidler's "characteristic points", which means something.

Another point of contact between the authors' method and those that have preceded, is the "pennant diagram", which has heretofore been known as "Ostenfeld's auxiliary diagram", as used in connection with Fidler's method. This diagram is described in the paper by Messrs. Richart and Wilson previously referred to, who state in a footnote that "Professor Ostenfeld devised this form of diagram in making a very general application of Fidler's method to various forms of continuous beams". The "pennant diagram" is identical with Ostenfeld's, both in appearance and function. The only point of departure is that under the latter name it was regarded merely as an adjunct to a major principle, whereas under the name of "pennant" it becomes a "prime mover", since it (with a little assistance from the *U* and *V*-points) determines the "conjugate points" on which the whole method hinges. Ostenfeld's diagram determines these same points, by what name if any, the writer does not know, but as a consequence and not as a cause.

It will be seen that in all essentials the authors' applied method is identical with that of Fidler plus Ostenfeld; the difference being that the authors start in with the effect, and, casually acquiring the "characteristic points", all unknown as such, at some stage of the process, prove the cause in terms of *x*, *y*, and *z*, *so* to speak, whereas, in Fidler's theory, this process is reversed, to its great improvement and clarity.

By definition, Fidler's "characteristic points" are such that the distance from the points to the moment closing line is a function of the slope of the beam at the support, and this he proved to hold true for the points when located according to the principles developed. Having these points, the solution of the problem consists simply in drawing moment closing lines that satisfy the requirements for distance from the points. This can be done by trial with relative ease if no other means are presented; that is, the solution is not dependent on "pennant diagrams", "conjugate points", or any other extraneous conception. Of course, a direct solution is preferable to one by trial, and this Ostenfeld's diagram provides. It is, however, nothing but a special form of an auxiliary geometrical construction that locates a "conjugate" point through which any line passing will satisfy the conditions imposed by Fidler's points. The same definite result may be accomplished by several other constructions, one of which will be given subsequently. Certainly, the conception of conjugate points is in no way essential to the problem, for the writer has been using them for several years without ever being impressed with the fact that they required a name and theory to support them. The purpose was equally well served when they were merely auxiliary points on the moment closing line.

In general, it appears to the writer that the authors' method of analysis is a clear case of "the tail wagging the dog"; that Fidler's points, by whatever name the authors may call them, are necessarily the actuating principle under

cover of a mass of more or less incidental theory which principally serves to prove the correctness of the points. Furthermore, when Fidler's points are adopted as the fundamental argument, there is available at once a definite, tangible, and self-sufficient law, which, if expressed in its general terms, applies without change to all the cases that the authors have considered. Also, as will be shown, the conception of the "characteristic points" leads directly to some very material short cuts, as compared with the authors' methods. It is for these reasons that the writer believes the purpose of the paper would have been better served had the authors informed themselves of Fidler's principles and used them as the foundation. This would have detracted nothing from the value and timeliness of the paper, for the strangest thing about Fidler's method, in view of its excellence, antiquity, and rather broad publication, is that so few engineers (relatively) are acquainted with it. Furthermore, the original material is generally difficult to obtain and to digest because of the peculiar form in which the subject is presented. Also, as far as the writer is aware, the method has never been carried to its logical and final development. Therefore, there was a real need for a paper such as the authors have presented, and this is in no wise diminished by the fact that the presentation would involve little of true originality or novelty.

Another peculiar coincidence as to methods and results is to be found in the tables of constants (Section 14*), for cases of varying moments of inertia. A very comprehensive set of tables of identical form, but different values, has been prepared and published by A. Strassner,† who developed a system quite different from that of Fidler. Superficially, there would appear to be no connection between the ideas and no possibility of using Strassner's tables in connection with Fidler's method. Certainly, Strassner himself indicates no such possibility; but due to the ingenuity of Walter H. Ruppel, Assoc. M. Am. Soc. C. E., assisted by Mr. Herman Schorer, both of whom were then (middle of 1924) associated with the writer on some rather intricate design, it was discovered that Strassner's tables could be adapted with relative ease to give the heights and lateral positions of Fidler's "characteristic points". Incidentally, these tables also give the fixing moments at the ends of the beam, as required in the solution of problems by the slope-deflection method. Mr. Ruppel worked out a complete set of formulas for transposing Strassner's tabular values into the quantities required in Fidler's or the slope-deflection method, an entirely original and very creditable achievement.

To return to the aforementioned similarity, the two tables given by the authors are identical in form, and even to interval of the arguments, with certain of Strassner's tables. The quantities which the authors denote as u and m, Strassner lists as, say, r and w, and the values are not alike. As Mr. Ruppel discovered, however, $u = \frac{w}{2\,r + w}$, and $m = \frac{2\,r + w}{6\,I}$; and this is verified by the authors' tables even to the last decimal place. The series of coincidences represented here is little short of marvellous.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1618.

† "Neuere Methoden," Vol. 1, 2d Edition, "Der durchlaufende Rahmen."

The authors refer very casually to the preparation of these tables, but the writer has no desire to undertake such a task. The time and effort required for original computations would be appalling, and out of all proportion to their value to the individual designing engineer. An alternative would be to accept Strassner's tables and modify them according to Mr. Ruppel's findings, which in itself is no mean undertaking. Fortunately, Mr. Ruppel has done this work and will present it in a discussion of the paper,* so that each engineer will not be faced with the painful necessity of preparing his own original tables, as suggested by the authors.

Fidler's Method Needs Explanation.—As the writer has intimated, he is of the opinion that Fidler's Method of Characteristic Points is the peer of all methods for treating moments in restrained and continuous beams, and that it deserves a wider publicity and understanding than it now has. Unfortunately, as far as the writer is aware, there is no really acceptable and comprehensible treatment of the subject to which those interested might be referred. There are Fidler's own writings, of course, but these are not in current circulation and therefore are difficult to obtain. Although the writer has not read these books and so cannot speak from personal knowledge, he is informed, by engineers well qualified to judge, that the subject is there presented in such a peculiar and awkward manner as to be unnecessarily difficult to follow and digest. Furthermore, Fidler did not carry the development of his formulas and methods to cover conditions which now obtain in structural design, so that judging from his own treatment the method, might be thought inadequate to meet the complex requirements of to-day. Aside from Fidler's own writings, the discussions of the subject within the writer's knowledge, are exceedingly fragmentary and treat only of simple cases, without proof. The only one that has come to the writer's personal attention is the brief abstract by Richart and Wilson, previously referred to. It appears evident that the subject needs to be presented anew and in a comprehensive manner adapted to present-day uses.

Using Fidler's fundamental proposition as the argument, the writer has expanded his method to include the most general cases, and has shown that the basic equation controls throughout in unchanged form. In view of the situation as outlined, it seems appropriate to give here a complete description of this development. It is hoped that the presentation will succeed in demonstrating the inherent simplicity and excellence of Fidler's Method of Characteristic Points, and of making this method more generally available to the profession.

In so far as the writer is personally concerned, this discussion and the method of development is entirely original, but no claim is laid to the fundamental ideas on which it is based. As far as he is aware, Fidler's method has not heretofore been developed to cover the more complex cases here treated, in which case the presentation will be fully justified. In any event it is certain that these broader applications of the method are not generally known.

* It is expected that the discussion by Mr. Ruppel will be published in a subsequent number of *Proceedings*.

Definitions and Conditions for Fidler's "Characteristic Points."—Before entering on the general equations relating to the "characteristic points", it will be advisable to discuss their more evident and tangible properties, particularly since fundamentally there is nothing to indicate the existence of such points, and the demonstration that they do exist must take the form of proving that properties observed for specific cases are also of general application. The conception of the characteristic point in a general sense, is a mere hypothesis, and it must be demonstrated that this hypothesis is, in fact, a statement of the law of structural mechanics.

In a bending moment diagram, let the line connecting the plotted moments over the supports be called the moment closing line. Then, the "characteristic point" may be defined as a point such that the vertical intercept between the point and the moment closing line is a measure of and proportional to the slope of the beam over the support.

By the slope of the beam at or over the supports is meant the angular change in the direction of the elastic curve due to the stress in the beam. When the end of a beam is fully restrained, or "fixed," this angular change is zero by definition. Since the vertical distance between the characteristic point and the moment closing line is proportional to the angular change at the end of the beam, and since this angular change is zero when the end is fixed, the distance from the point to the line must also be zero; that is, the point lies on the line for this condition. Since, in general, the slopes at the two ends of the beam may be different, it is evident that there must be one characteristic point for each end of the beam. By definition, then, these two characteristic points will lie on the moment closing line when the two ends of the beam are fixed. That this conception has an important bearing on the practical application of the method will be shown subsequently. Likewise, by definition, considering the same beam under the same loading, but fixed at one end only and freely supported at the other, the characteristic point corresponding to the fixed end must lie on the moment closing line. The tendency to angular change at the ends of a beam is a function of both the loading and the properties of the beam itself, and since the position of the characteristic point is a function of the angular change this position must likewise be a function of the loading and beam properties. Keeping the beam and loading constant, it appears probable that, corresponding to the end of the beam considered fixed, there will be a characteristic point common to the two conditions considered. If so, it follows that this point must lie at the intersection of the corresponding moment closing lines. In fact, if there is a characteristic point, it can be nowhere else since no other position would satisfy the conditions imposed.

Consider now a beam of constant, $E I$, with its ends fixed, and under uniform load. The end moments are $\frac{w L^2}{12}$. If one end only is fixed, the end moment is $\frac{w L^2}{8}$, or 1.5 times the preceding value. The moment diagrams, superimposed, are shown in Fig. 51, for negative moments only.

The line, $B' C'$, is the moment closing line when the beam is fixed at both ends, for which, let $M' = M'' = 1$. The lines, $B'' C$ and $B C''$, are the corres-

ponding lines when the beam is fixed alternately at the one end and simply supported at the other, for which, M' or $M'' = 1.5$. It is evident that the line, $B'C'$, intersects the line, BC'', at a point, V, which by proportion is at two-thirds the height of the triangle, BCC'', and, therefore, at a horizontal distance of $\frac{1}{3}L$ from C. Likewise, U is a point $\frac{1}{3}L$ distant from B. Furthermore, it will be observed that the points, U and V, are in the same vertical plane as the centers of gravity of the corresponding triangular end moment areas. Also, when EI is constant, the $\frac{M}{EI}$-areas corresponding to the two end moments are likewise triangular, and their centers of gravity lie on the same vertical as those of the M-areas; therefore, in the case considered, on the same verticals as the points, U and V, which satisfy the conditions imposed for characteristic points.

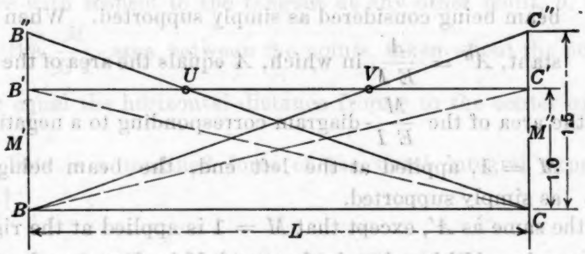

FIG. 51.

From the foregoing, certain general conditions have been established which the characteristic points must satisfy, and characteristic points have been established to satisfy a specific condition. Since there can necessarily be only one position of each characteristic point corresponding to a certain beam and loading, it follows that, if the hypothesis regarding the points is true, the general case must conform to the specific facts developed. It remains to demonstrate that such is the case; that a point located on the moment closing line, vertically above the center of gravity of the negative $\frac{M}{EI}$-area corresponding to one end of the beam, satisfies the definition of the characteristic point for that end.

Notation and Theorems.—The notation herein will be the same as that used by the authors, with the additions noted. It is assumed, unless otherwise stated, that the beam supports are at the same elevation and that the beam is horizontal. Also, that the moment diagrams are plotted with respect to a horizontal line between supports, which line will be called the "moment reference line" (sometimes called the "M. R. L."). It is also assumed that both positive and negative moments will be plotted normal to and above this line, as customary.

Let l = the span of the beam, center to center of supports.

U = the left characteristic point, or its height above the moment reference line (see Fig. 53).

V = the right characteristic point, or its height above the moment reference line (see Fig. 53).

U' and V' = points on the moment closing line, vertically above U and V, respectively. (It will be observed that U and V are the particular values of U' and V' when the moment closing line is that for the beam completely fixed at both ends.) The "moment closing line" will sometimes be abbreviated to "M. C. L."

d' = the vertical distance from the moment closing line to the left characteristic point, $= U—U'$.

d'' = the corresponding distance on the right side, $= V—V'$.

M' = the restraining moment at the left support.

M'' = the restraining moment at the right support. (Restraining moments will also be referred to as negative moments.)

A^0 = the area of the $\dfrac{M}{E\,I}$-diagram corresponding to the actual load, the beam being considered as simply supported. When $E\,I$ is constant, $A^0 = \dfrac{A}{E\,I}$, in which, A equals the area of the M-diagram.

A' = the area of the $\dfrac{M}{E\,I}$-diagram corresponding to a negative moment, $M = 1$, applied at the left end, the beam being considered as simply supported.

A'' = the same as A', except that $M = 1$ is applied at the right end.

(In the foregoing, if l is taken in feet, and M in foot-pounds, then E is in pounds per square foot and I is in feet to the fourth power.)

x^0 = the horizontal distance from the left support to the center of gravity of A^0.

x' = the horizontal distance from the left support to the center of gravity of A'.

x'' = the horizontal distance from the left support to the center of gravity of A''.

ϕ' = the slope of the beam at the left support.

ϕ'' = the slope of the beam at the right support. (By slope of beam is meant the angle, expressed in radians, through which the neutral axis is turned due to the stress in the beam, and is here considered as the angle which the tangent to the elastic curve makes with the horizontal.)

θ = the angle between the tangents to any two points on the elastic curve.

D = the deflection of any point on the elastic curve with respect to the position of the point in the unstressed beam; that is, the vertical displacement of the point due to stress.

y' = the vertical distance between a point on the elastic curve and the tangent to any other point on the curve.

The following theorems are used as the basis of the subsequent demonstration. These theorems are in no sense original, but they and their proofs

are here given because they have not generally been developed or used in American textbooks; they are given in one group for ease of reference, and because they are more or less interdependent.

Theorem No. 1.—The angle, θ, between the tangents to any two points on the elastic curve, is the $\frac{M}{E\,I}$-area between the points. Let the $\frac{M}{E\,I}$-area between the points be called a. Then, $\theta = a$.

Proof: This is simply a modified form of the usual integral expression as given in practically any text on the mechanics of materials, in the form of $\theta = \int \frac{M\,d\,x}{E\,I}$.*

Theorem No. 2.—The vertical displacement, y', of any point, c, on the elastic curve with respect to the tangent at any other point, b, is the statical moment of the $\frac{M}{E\,I}$-area between the points, taken about the point, c. That is, letting z equal the horizontal distance from c to the center of gravity of a, $y' = az$.

Proof: This is a modified form of the usual integral expression, $y' = \int \frac{M\,x\,d\,x}{E\,I}$.†

Theorem No. 3.—The angle, ϕ, which the elastic curve makes with the horizontal at the support is the reaction at that support when the beam is considered as a simple span supporting the $\frac{M}{E\,I}$-area treated as a load. That is, $\phi' = \frac{A^0\,(L - x^0)}{L}$, and $\phi'' = \frac{A^0\,x^0}{L}$.

Proof: (See Fig. 52.) Given a loaded beam, $B\,C$, the angle at B is ϕ', and from Theorem No. 2 the y' of C with respect to the tangent at B equals $A^0\,z = A^0\,(L - x^0)$. From the diagram, however, $y' = \phi'\,L$, also. Therefore, $\phi'\,L = A^0\,(L - x^0)$, and, hence, $\phi' = \frac{A^0\,(L - x^0)}{L}$, which is the expression for the reaction at B when the beam is loaded with the area, A^0.

Theorem No. 4.—The deflection, D, of any point, c, on the elastic curve, is the bending moment at the point when the beam is considered as a simple span supporting the $\frac{M}{E\,I}$-area treated as a load. That is, if the distance of c from the left support is called x, then, $D = \left[\frac{A^0\,(L - x^0)}{L}\right] x - a\,z = \phi'\,x - a\,z$, in which, a is the $\frac{M}{E\,I}$-area between the left support and the point, c, and z is the distance from c to the center of gravity of a.

Proof: (See Fig. 52.) The reaction at B is ϕ', from Theorem No. 3. From the diagram, the deflection, D, at c = distance $c\,e$ = distance $e\,f$ — distance $c\,f$;

* For proof, if any is required, see Church's "Mechanics of Materials."

† See Church's "Mechanics of Materials."

but distance $ef = \phi' x$, and from Theorem No. 2, distance $cf = az$. Therefore, $D = \phi' x - az$, and since ϕ' is the reaction at B, this is the expression for the moment at c when the beam is loaded with the area, A^0.

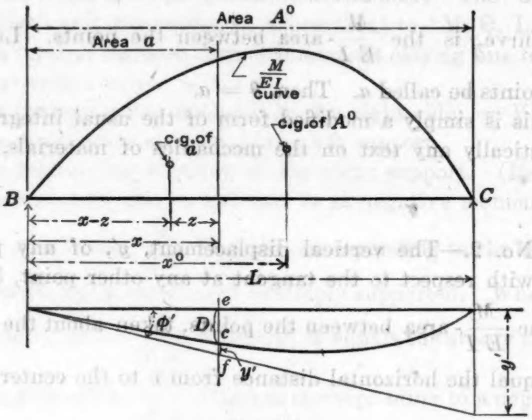

FIG. 52.

Theorem No. 5.—The angle, ϕ', at the left support, B, of a beam, due to a certain moment, M, applied at the right support, C, is equal to the angle, ϕ'', at C, when the same M is applied at B.

Proof: Consider the $\frac{M}{E\,I}$-area, A'. The moment at any point is equal to $\frac{(L - x)}{L}$, since $M = 1$ by definition, and x is measured from the left support. Corresponding to any value of x there is some particular value of I. Accordingly, I may be regarded as a function of x, and expressed as $f(x)$. Then,

$$A' = \frac{I}{E} \int_B^C \frac{\frac{L - x}{L}}{f(x)} d\,x$$

and from Theorem No. 2 the displacement of B with respect to the tangent at C is,

$$y' = \frac{I}{E} \int_B^C \frac{L - x}{L\,f(x)} x\,d\,x$$

but the angle at C equals

$$\phi'' = \frac{y'}{L} = \frac{I}{E} \int_B^C \frac{(L - x)\,x}{L^2\,f(x)} d\,x \quad \text{.................(48)}$$

Consider now the $\frac{M}{E\,I}$-area, A''. The moment at any point is $\frac{x}{L}$, and ,

$$A'' = \frac{I}{E} \int_C^B \frac{\frac{x}{L}}{f(x)} d\,x$$

The displacement of C with respect to the tangent at B equals

$$y'' = \frac{I}{E} \int_C^B \frac{x}{L\,f(x)} (L - x)\,d\,x$$

but the angle at B equals

$$\phi' = \frac{y''}{L} = \frac{I}{E} \int_C^B \frac{x\,(L - x)}{L^2\, f\,(x)}\, d\,x \ldots\ldots\ldots\ldots\ldots (49)$$

Equation (49) is identical with Equation (48), except as to sign, due to change of limits. Therefore, $\phi'' = -\phi'$ when the moment, $M = 1$, is alternately applied at B and C, respectively.

General Proof of the Theorem of "Characteristic Points."—Consider a beam of such character that $E\,I$ is a variable throughout. The ends are restrained by the moments, M' and M''. The values of A' and A'' are different; x' and x'' are apparently unrelated and may vary over wide limits. The conditions may be assumed as represented in Fig. 53. This drawing is purely diagrammatic and no attempt is made at consistency. The areas, A°, A' and A'', may take practically any form, depending on the shape of the member and the loading. The magnitude of the negative $\frac{M}{E\,I}$ - loadings is $A'\ M'$ and $A''\ M''$, and the lateral position of the centers of gravity of these areas is the same as for A' and A''. These negative areas may be regarded as upward loads, and hence are shown below the line, opposite the positive $\frac{M}{E\,I}$ - area.

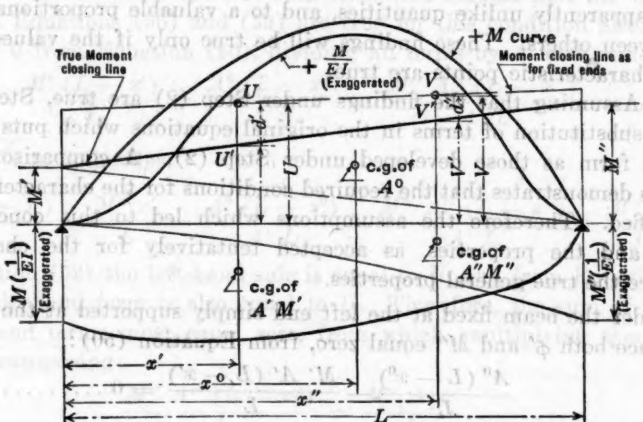

FIG. 53.

From Theorem No. 3,

$$\phi' = \frac{A^0\,(L - x^0)}{L} - \frac{M'\,A'\,(L - x')}{L} - \frac{M''\,A''\,(L - x'')}{L} \ldots\ldots (50)$$

$$\phi'' = \frac{A^0\,x^0}{L} - \frac{M'\,A'\,x'}{L} - \frac{M''\,A''\,x''}{L} \ldots\ldots\ldots\ldots\ldots (51)$$

From the geometry of Fig. 53:

$$U' \text{ (or } U) = \frac{M'\,(L - x')}{L} + \frac{M''\,x'}{L} \ldots\ldots\ldots\ldots\ldots (52)$$

$$V' \text{ (or } V) = \frac{M'\,(L - x'')}{L} + \frac{M''\,x''}{L} \ldots\ldots\ldots\ldots\ldots (53)$$

These are the four fundamental equations on which the following demonstration will be based.

The proof that the theorem of characteristic points applies to the general case is necessarily quite involved and peculiar. It is described in some detail, as otherwise the significance of the following four steps that are required will not be apparent.

(1).—It will be assumed that the theorem of "characteristic points" is still only a hypothesis, and that there are only those indications relative to the points which have so far been developed in this discussion. It will be further assumed that the hypothesis is true, and that the points will lie on the moment closing line as for completely fixed ends, and vertically above the center of gravity of the areas, A' and A'', respectively, as heretofore indicated.

(2).—A value will be found for the height of each characteristic point for a specific case that will permit this determination. If the theorem is true, these values must be the true values, since each point can have only one value for given conditions of beam and load.

(3).—The values as found will then be assumed to be true, and be substituted in a specific case involving both, which will lead to an identity between apparently unlike quantities, and to a valuable proportional relationship between others. These findings will be true only if the values as found for the characteristic points are true.

(4).—Assuming that the findings under Step (2) are true, Step (3) permits the substitution of terms in the original equations which puts them into the same form as those developed under Step (2). A comparison of these equations demonstrates that the required conditions for the characteristic point are satisfied. Therefore the assumptions which led to this conclusion are verified, and the properties, as accepted tentatively for the characteristic points, are the true general properties.

Consider the beam fixed at the left end simply supported at the right end. Then, since both ϕ' and M'' equal zero, from Equation (50):

$$\frac{A^0 \, (L - x^0)}{L} - \frac{M' \, A' \, (L - x')}{L} = 0 \quad \text{.............. (54)}$$

From Equation (52):

$$U = \frac{M' \, (L - x')}{L} \quad \text{.......................... (55)}$$

From Equation (54):

$$\frac{M' \, (L - x')}{L} = \frac{A^0 \, (L - x^0)}{L \, A'}$$

and the left-hand term of this equation equals the right-hand term of Equation (55). From this:

$$U = \frac{A^0 \, (L - x^0)}{L \, A'} \quad \text{.......................... (56)}$$

which will be accepted tentatively as true. It is not subject to proof at this stage.

Consider, now, the conditions as to end supports reversed, in which case, ϕ'' and M' are both equal to zero, and, from Equation (51),

$$\frac{A^0\, x^0}{L} - \frac{M''\, A''\, x''}{L} = 0 \qquad (57)$$

and from Equation (53):

$$V = \frac{M''\, x''}{L} \qquad (58)$$

From Equation (57):

$$\frac{M''\, x''}{L} = \frac{A^0\, x^0}{L\, A''}$$

from which, by comparison with Equation (58),

$$V = \frac{A^0\, x^0}{L\, A''} \qquad (59)$$

which also will be tentatively accepted as true.

To demonstrate that the values of U and V as given by Equations (56) and (59) are true, it will be necessary to prove that these values satisfy the general conditions imposed.

Now, consider the beam fixed at both ends, and assume that the values of U and V in Equations (56) and (59) are true for this condition also. Since $\phi'' = \phi' = 0$ from Equation (50), dividing all terms by A' and transposing:

$$\frac{M'\,(L - x')}{L} = \frac{A^0\,(L - x^0)}{A'\, L} - \frac{M''\, A''\,(L - x'')}{A'\, L} \qquad (60)$$

Adding $\dfrac{M''\, x'}{L}$ to both sides,

$$\frac{M'\,(L - x')}{L} + \frac{M''\, x'}{L} = \frac{A^0\,(L - x^0)}{A'\, L} - \frac{M''\, A''\,(L - x'')}{A'\, L} + \frac{M''\, x'}{L} \qquad (61)$$

From Equation (52) the left-hand side is equal to U, and from Equation (56) the first right-hand term is also equal to U. Therefore, the sum of the last two right-hand terms must equal zero, from which, multiplying these terms by A' and transposing:

$$\frac{M''\, A''\,(L - x'')}{L} = \frac{M''\, A'\, x'}{L} \qquad (62)$$

Substituting in Equation (50), the general equation becomes:

$$\phi' = \frac{A^0\,(L - x^0)}{L} - \frac{M'\, A'\,(L - x')}{L} - \frac{M''\, A'\, x'}{L} \qquad (63)$$

Now, $d' = U - U'$. Therefore, from Equations (56) and (52),

$$d' = \frac{A^0\,(L - x^0)}{L\, A'} - \frac{M'\,(L - x')}{L} - \frac{M''\, x'}{L} \qquad (64)$$

As Equation (64) is Equation (63) divided by A',

$$d' = \frac{\phi'}{A'}, \text{ or } \phi' = d'\, A' \qquad (65)$$

This shows that the distance from the assumed characteristic point to the moment closing line is a function of and proportional to the angle at the

adjacent support, which satisfies the definition of the characteristic point. Therefore, the assumed properties of the point are the true properties, and the theorem is verified for the point, U.

Considering now the point, V, from Equation (51), dividing all terms by A'' and transposing,

$$\frac{M'' x''}{L} = \frac{A^0 x^0}{L A''} - \frac{M' A' x'}{L A''} \qquad (66)$$

Adding $\frac{M' (L - x'')}{L}$ to both sides,

$$\frac{M'' x''}{L} + \frac{M' (L - x'')}{L} = \frac{A^0 x^0}{L A''} - \frac{M' A' x'}{L A''} + \frac{M' (L - x'')}{L} \qquad (67)$$

From Equation (53) the left-hand expression equals V, and from Equation (59) the first term of the right-hand side also equals V. Therefore, the sum of the last two right-hand terms equals zero. Multiplying each of these terms by A'' and transposing,

$$\frac{M' A' x'}{L} = \frac{M' A'' (L - x'')}{L} \qquad (68)$$

Substituting in Equation (51), the general equation becomes:

$$\phi'' = \frac{A^0 x^0}{L} - \frac{M' A'' (L - x'')}{L} - \frac{M'' A'' x''}{L} \qquad (69)$$

Now, $d'' = V - V'$. Therefore, from Equations (59) and (53),

$$d'' = \frac{A^0 x^0}{A'' L} - \frac{M' (L - x'')}{L} - \frac{M'' x''}{L} \qquad (70)$$

As Equation (70) is Equation (69) divided by A'',

$$d'' = \frac{\phi''}{A''}, \text{ or } \phi'' = d'' A'' \qquad (71)$$

This gives the same proof for the point, V, as Equation (65) gave for the point, U, as noted previously. The theorem of characteristic points is thereby fully verified for the general case.

Dividing both sides of Equation (62) by M'' and multiplying by L,

$$A'' (L - x'') = A'' x' \qquad (72)$$

As $L - x''$ is the distance from the right end of the span to the center of gravity of A'', and x' is the distance from the left end to the center of gravity of A', Equation (72) gives the useful principle that the areas, A' and A'', are inversely proportional to the distances of their centers of gravity from the ends of the diagrams used as origins of moments. This provides an easy means of finding any one of four required values when the other three are known, or it provides a check on the accuracy of the results if the four values are determined independently.

The foregoing relationship was determined incidentally and was predicated on the truth of apparently unrelated assumptions. The proposition may be proved otherwise as follows:

From Theorem No. 5, $\phi' = \phi''$ when the moments, $M'' = 1$ and $M' = 1$, are applied alternately at the right and left ends, respectively, of the beam. Since ϕ' is the angle corresponding to M'', it is the left-end reaction of the

$\frac{M}{E\,I}$-area, A'', and for a like reason, ϕ'' is the right-end reaction of A'. Therefore, $\phi' = \frac{A''\,(L - x'')}{L}$ and $\phi'' = \frac{A'\,x'}{L}$. Since $\phi' = \phi''$, $A''\,(L - x'') = A'\,x'$, which is an independent verification of the conclusions drawn from Equation (61), and hence of the assumptions on which these conclusions were predicated.

To recapitulate, the following facts have been demonstrated:

(a) The theorem of Fidler's characteristic points is true for all conditions of beam and load.

(b) These points are located vertically above the centers of gravity of the areas, A' and A'', respectively.

(c) The height of the points is given by the following: $U = \frac{A^0\,(L - x^0)}{L\,A'}$ (Equation (56)), and $V = \frac{A^0\,x^0}{L\,A''}$ (Equation (59)), and these are points on the moment closing line, considering the beam as fixed at both ends.

(d) For any condition of end restraint, the vertical distances between the characteristic points and the moment closing line are: $d' = \frac{\phi'}{A'}$ (Equation (65)), and $d'' = \frac{\phi''}{A''}$ (Equation (71)), which show that these distances vary directly as the slope of the beam at the adjacent support, and inversely as A' or A''.

(e) The products, $A'\,x'$ and $A''\,(L - x'')$, are equal, or $A' : A'' = (L - x'') : x'$.

It is not required to evaluate d' or d'', as indicated in Equations (65) and (71). These equations have served their purpose in defining the fundamental relationships and proportions. By applying this law in the graphical construction to follow, the values of d' and d'' are automatically determined. Their actual values, however, are unimportant, since only their relative values enter into the solution of the problem.

It will be observed that, other things being equal, the height of the characteristic points is a function of the magnitude of the load on the span, and that their lateral position is a function of $\frac{1}{E\,I}$, and hence depends solely on the shape of the beam if E is constant. From this it will be seen that the "characteristic points" are indeed well named, being characteristic both of the loading on the beam and of the qualities of the beam itself.

"Characteristic Points" for Specific Conditions.—If the member is of varying $E\,I$, but symmetrical about its middle, $x' = L - x''$, and $A' = A''$. The characteristic points will then be symmetrically located on the span. If the load is also symmetrical about the center of the span, $U = V$, since $L - x^0 = x^0$.

If the member is of constant $E\,I$, the $\frac{M}{E\,I}$-areas are triangular and $A' = A'' = \frac{L}{2\,E\,I}$. Since the characteristic points are on the same vertical as the center

of gravity of these areas, $x' = \frac{L}{3}$ and $x'' = \frac{2\,L}{3}$. Also, substituting the values of A' and A'' in Equations (56) and (59),

$$U = \frac{2\,A\,(L - x^0)}{L^2} \dots\dots\dots\dots\dots\dots\dots (56a)$$

and

$$V = \frac{2\,A\,x^0}{L^2} \dots\dots\dots\dots\dots\dots\dots (59a)$$

When, in addition, the load is symmetrical, these reduce to

$$U = V = \frac{A}{L} \dots\dots\dots\dots\dots\dots\dots (56a)$$

"Characteristic Points" for Beams of Constant $E\,I$, from the End That Fixes the Moments.—Except in cases of very simple loading the determination of the value of A, and of x^0 if the load is unsymmetrical, would require a considerable expenditure of time and effort. Fortunately, this usually can be avoided by utilizing the principle that the U and V-points lie on the moment closing line as for the beam with both ends fixed. Let M' equal the moment at the left end, and M'' that at the right end for this condition. Then

$$U = M' + \frac{(M'' - M')}{3}$$

and

$$V = M' + \frac{2\,(M'' - M')}{3}$$

Most engineering handbooks contain tables of end moments for fixed beams under various arrangements of load, from which, for most cases, U and V may be derived readily. A very comprehensive set of such tables appears in "Stresses in Framed Structures", Hool and Kinne, pages 486 to 490, inclusive, from which the moments, M' and M'', may be determined with relative ease for almost any type of loading. In these tables, $C_{AB} = M'$ and $C_{BA} = M''$. This suggested method of determining U and V is predicated, of course, on the possession of such tables and their application to the problem at hand. Otherwise the suggestion does not apply, since it would be more difficult to find the values of M' and M'' than to determine A and x^0. Likewise, it applies only when the member is of constant $E\,I$, since such tables do not cover any other case.

In these tables (Hool and Kinne) are also given values of H_{AB} and H_{BA}. The former is the moment at the left end when that end is fixed and the right end simply supported; the latter is the moment at the right end for the reverse condition. Consequently, $U = \frac{2}{3}\,H_{AB}$, and $V = \frac{2}{3}\,H_{BA}$. The values are the same as those just given, of course, and are somewhat easier to compute.

"Characteristic Points" for General Case.—If modified Strassner's tables are available, as the writer understands they will be, the values of U, V, x' and x'' may be derived with ease for beams of any shape and loaded in any

manner. The presentation of this phase is, however, outside the scope of this discussion.

Should such tables as described be not available, it will be necessary in all complex cases to determine the values of A^0, A', A'', x^0, x', and x'', in order to compute U and V. No great accuracy is required in this evaluation, and the graphical method will give a satisfactory solution. If the $\frac{M}{E\,I}$-diagrams are plotted to a convenient scale on a good quality of light cardboard or press-board, the areas may first be taken off with planimeter; then the diagrams may be cut out and their centers of gravity easily found by the method of successive suspensions, giving two intersecting lines each containing the center of gravity, which therefore will lie at the intersection.

Graphical Application of Theorem of Characteristic Points and Auxiliary Constructions.—It remains to make application of the foregoing facts in the graphical solution of moment distribution in continuous beams. Referring to Fig. 54, let V and U be the two characteristic points adjacent to and on opposite sides of one of the supports of a continuous beam. Let A'', d'', and ϕ'' refer to the right-hand properties of the left-hand span, and let A', d', and ϕ' refer to the left-hand properties of the right-hand span, according to the adopted notation.

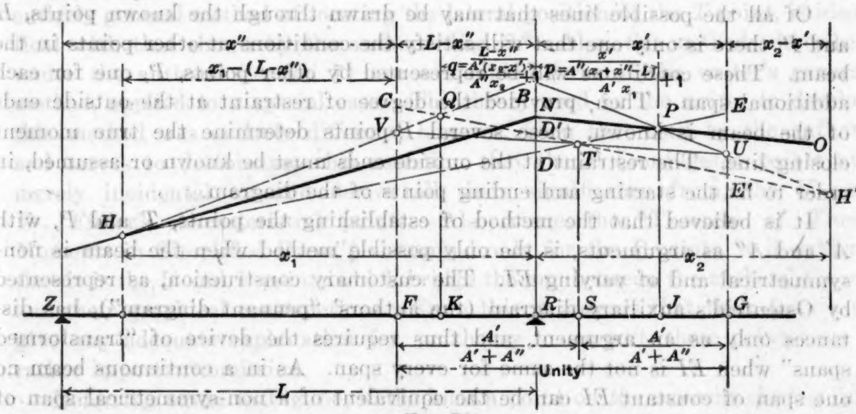

FIG. 54.

The slope of the beam over the support must be common to both spans; that is, $\phi'' = -\phi'$ or *vice versa* with respect to sign, since one span will slope upward and the other downward, at the support. From Equation (71), $\phi'' = d''\,A''$, and from Equation (65), $\phi' = d'\,A'$; but since $\phi'' = -\phi'$ $d''\,A'' = -\,d'\,A'$, from which, $d'' : A' = -\,d' : A''$. That is, considering the two characteristic points adjacent to and on opposite sides of a support, one of these is above and the other below the moment closing line, and the distances between the points and the line are inversely proportional to the areas, A' and A'', corresponding to the points. It is required to develop a graphical construction that will satisfy this condition.

In Fig. 54, let H be a known point on the moment closing line. Draw the line, $HVBU$. This line satisfies the required conditions, since it contains the

point, H, and $d'' = d' = 0$. (Because, if $d'' = 0$, $d'' A'' = 0$, and, hence, $- d' A' = 0$; but, as A' is not zero, $- d'$ equals zero.)

Draw the line, VU. It is required to establish a point on this line such that any other line drawn through it will satisfy the required proportions of d'' to d'. It has been shown that d'' is proportional to A', and d' to A''. Therefore, SF is proportional to A' and SG to A'', from which $SF : A' = FG : A' + A''$. Let $FG = 1$. Then, $SF = \frac{A'}{A' + A''}$. Similarly, $SG = \frac{A''}{A' + A''}$. Establish the point, S, accordingly and erect a vertical to intersect UV at the point, T, which is the required point, as found previously.

Draw the line, HTE, which satisfies the requirements, since it contains the point, H, and passes through T. This line intersects the line, BU, in the point, P.

Now, the distance, VC, is proportional to d'' and EU is proportional to d'; also, both VC and EU are proportional to BD. From the relationships of the triangles involved, it is apparent that any other line, such as ONH, passing through P and H, will maintain the same proportional values of d'' and d' as those given by the line, HTE. Therefore, the point, P, is a point on the moment closing line, since any line through this point will satisfy the required conditions thus far imposed.

Of all the possible lines that may be drawn through the known points, H and P, there is only one that will satisfy the conditions at other points in the beam. These conditions will be represented by other points, P, one for each additional span. Then, provided the degree of restraint at the outside ends of the beam is known, these several P-points determine the true moment closing line. The restraint at the outside ends must be known or assumed, in order to fix the starting and ending points of the diagram.

It is believed that the method of establishing the points, T and P, with A' and A'' as arguments, is the only possible method when the beam is non-symmetrical and of varying EI. The customary construction, as represented by Ostenfeld's auxiliary diagram (the authors' "pennant diagram"), has distances only as an argument, and thus requires the device of "transformed spans" when EI is not the same for every span. As in a continuous beam no one span of constant EI can be the equivalent of a non-symmetrical span of varying EI such diagrams are not applicable to this case. It may be stated in passing that such diagrams can be applied when the spans are of varying EI within themselves, but symmetrical, if the spans are laid off proportionally to their $A' = A''$. It would seem, however, that there is no merit in using the principle of "transformed spans" when the same result can be accomplished in a more simple and direct fashion.

A noteworthy feature of the foregoing method of establishing the point, T, with A' and A'' as the argument, is that it makes the construction entirely independent of the scale length of the spans. These may be laid out to any distance, regardless of their true relative lengths. Then, provided the characteristic points are located in their true relative position on the span and that A' and A'' are proportional to the true span lengths, the construction will give the correct results. No proof of this statement will be given

here, as it is practically self-evident, and can be easily verified by actual construction.

Let L_1 = the span to the left, and L_2 = the span to the right of the support. When EI is different for these spans, but constant throughout each, $A'' = \frac{L_1}{2\,E_1\,I_1}$ and $A' = \frac{L_2}{2\,E_2\,I_2}$. If $E_1 = E_2$, as is customary, A'' may be taken as $\frac{L_1}{I_1}$, and A' as $\frac{L_2}{I_2}$, since only relative values of these areas are required.

When EI is constant throughout each span of the beam, A' varies as L_2 and A'' varies as L_1. Therefore, d'' and d' vary inversely as the spans in which they occur, or $SF : L_2 = SG : L_1$; from which, $SF : L_2 = FG : L_1 + L_2$. In this case, however, $FG = \frac{(L_1 + L_2)}{3}$, from which,

$$SF = \frac{(L_1 + L_2)\,L_2}{3\,(L_1 + L_2)} = \frac{1}{3}\,L_2.$$

Similarly, $SG = \frac{1}{3}\,L_1$, which explains the location of the "transposition line" in the customary diagrams.

It will be observed that the construction herein described, except for the entirely general method of locating the lateral position of the T-point, is identical with that shown by the authors' Fig. 8.* Also, the methods described under the authors' Section 7† are a modified form of the writer's general method, applied to a special case. The theory used by the writer to develop this method was entirely different, however, from that used by the authors and serves to demonstrate his assertion that the P (conjugate) points are merely incidental to, and derive all their significance from, the U and V- (Fidler's characteristic) points. Also, since the "Theorem of Three Moments in Simplified Form"‡ accounted only for the P-points, it is evident that it is the U and V-points which bridge the gap between the authors' theory and their graphical constructions. From this it appears that their failure to give any adequate explanation for, or theory to support, these points was a rather important omission.

Referring again to Fig. 54, if the known point on the moment closing line is H', to the right of the support instead of the left, a construction similar to that previously described will locate the point, Q, as indicated. This is also a point of the moment closing line, but in the span adjacent to the point, P.

A construction exactly as just described for locating the Q and P-points is inapplicable when the adjacent spans are unloaded. In an unloaded span, the U and V-points lie on the moment reference line, and hence will not generally permit the determination of the intersections required in the foregoing method. A modification of the construction to suit the condition of unloaded spans will now be developed.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1598.

† *Loc. cit.*, p. 1603.

‡ *Loc. cit.*, p. 1594.

As already shown, VC is proportional to A', and UE to A''. (See Fig. 54.) Since this construction deals with proportional relationships only, and VC may have any value, VC may be made equal to A', and UE equal to A''. Then, from the diagram,

$$BD = \frac{A'\,x_1}{x_1 + x'' - L}.$$

Therefore,

$$p : \frac{A'\,x'}{x_1 + x'' - L} = x' - p : A'',$$

from which,

$$p = \frac{x''}{\dfrac{A''\,(x_1 + x' - L)}{A'\,x_1} + 1}$$

Also,

$$BD' = \frac{A''\,x_2}{x_2 - x'}.$$

Therefore,

$$q : \frac{A''\,x_2}{x_2 - x'} = L - x'' - q : A',$$

from which,

$$q = \frac{L - x''}{\dfrac{A'\,(x_2 - x')}{A''\,x_2} + 1}$$

These expressions for p and q are too cumbersome for practical use, and the purpose can be accomplished more simply than by performing the evaluation. They are given here to demonstrate analytically certain important facts. It will be observed that the formulas contain no load terms, also, that the distances, p and q, determine the points, J and K, respectively. This shows that the lateral position of these latter points is independent of the load, being a function of the properties of the beam only; from which it follows that for any given beam, irrespective of the conditions of loading, the points, P and Q, will lie vertically above, or will be coincident with, certain points, J and K, as for the beam without load. When the points are coincident that point is also a point of inflection, since the moment closing line will cross there. If there is a point of inflection in an unloaded beam it will be either at K or J, depending on the position of the loaded span with respect to the unloaded span considered. For unloaded interior spans to the left of a single loaded span, the inflection point is at J; for similar spans to the right, it is at K, as will be apparent from the subsequent diagram.

It will be seen that the points, K and J, possess considerable significance within themselves. They are the fixed points of Ritter's and of Strassner's method, and are the principal arguments in their constructions. These points are of prime importance in considering moving loads or the construction of influence lines, and much time and labor will be saved by the knowledge that the moment closing line always passes through certain predetermined points

to the right and left of the load, irrespective of the position, character, and magnitude of the latter.

A graphical method will now be developed for determining the K and J-points. Referring to Fig. 54, consider that the load in the left-hand span is removed, and that the known point, H, lies on the moment reference line, $Z\,G$, as it generally will in practice. There being no load in the span the height of the characteristic points is zero; that is, V becomes coincident with F, and, hence, Q with K and B with R. The situation will be as indicated in Fig. 55 (a), the other points being determined by the method previously described. This diagram is not in any sense essential to the following demonstration, but is given to indicate the transition between the two constructions, and to show that these are essential in principle.

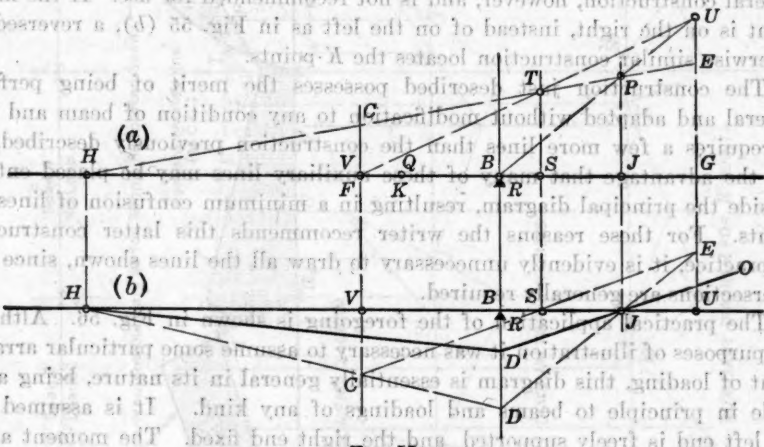

FIG. 55.

Consider now that the load in the right-hand span is also removed, there being some loaded span still farther to the right. Then, the points, T, P, and U, become coincident with S, J, and G, respectively, and all points in both spans lie in a straight line, as shown in Fig. 55 (b). Draw the verticals through the points, V and U. The point, S, being located as in Fig. 54, the line, $C\,E$, through this point will subtend on the above verticals the required proportional distances, $C\,V$ and $E\,U$. As proportionate values only are important, $V\,C$ may be made equal to A', and $E\,U$ equal to A''. Draw $H\,C$, produced, intersecting the vertical through the support at D, and draw $D\,E$, intersecting $R\,U$ in the point, J. Then $R\,D$ is proportional to $V\,C$; also to $E\,U$. As $V\,C = A'$ and $E\,U = A''$, $R\,D$ is proportional to both A' and A'', and it is evident from the triangles involved, that any other line as $O\,D'\,H$, through the point, J, will maintain the same proportional distances. Therefore, the point, J, is a point on the moment closing line, since any such line through it will satisfy the required proportions of d'' to d'. The point being also on the moment reference line, it is the inflection point for that span.

The point, S, is not essential to the location of J. Lay off $V\,C = A'$ and $E\,U = A''$. Produce $H\,C$ to D, and draw $D\,E$, intersecting $B\,U$ in the required

point, J. It is recommended, however, that the construction using the point, S, be adopted, thereby retaining the same methods throughout and also making the point available for other conditions.

With the exception of the manner of locating the point, S, the construction of Fig. 55 (b) is identical with the customary form of Ostenfeld's auxiliary diagram, which shows this diagram to be applicable to the general case, without transformation of spans, provided the point, S, is located with A' and A'' as arguments. It will be observed that this diagram establishes directly only the fixed point, J, but with the characteristic points as arguments, exactly as previously used to locate the point, P; also, that this latter point will lie at the intersection of a vertical through J with the line, $H\,T$, produced, or with the line, $B\,U$, as in Fig. 55 (a). This latter line is not adapted to the general construction, however, and is not recommended for use. If the known point is on the right, instead of on the left as in Fig. 55 (b), a reversed but otherwise similar construction locates the K-points.

The construction just described possesses the merit of being perfectly general and adapted without modification to any condition of beam and load. It requires a few more lines than the construction previously described, but has the advantage that many of these auxiliary lines may be placed entirely outside the principal diagram, resulting in a minimum confusion of lines and points. For these reasons the writer recommends this latter construction. In practice, it is evidently unnecessary to draw all the lines shown, since only intersections are generally required.

The practical application of the foregoing is shown in Fig. 56. Although for purposes of illustration it was necessary to assume some particular arrangement of loading, this diagram is essentially general in its nature, being applicable in principle to beams and loadings of any kind. It is assumed that the left end is freely supported, and the right end fixed. The moment at the left end being zero, the moment closing line will intersect the reference line at that end. The right end being fixed, the moment closing line will pass through the adjacent characteristic point, by definition.

The arrangement of Fig. 56 was chosen so as to show in one diagram, as far as possible, instances of every situation met in such problems. The auxiliary diagram for establishing the J-points is shown below and separate from the moment diagram, for the sake of clearness. It is not necessary to do this, but it will generally be advisable, in order to avoid a confusing jumble of lines and points.

The procedure in laying out such a diagram is, as follows:

1.—The moment reference and reaction lines are drawn, and the U and V-points located.

2.—The reference line for the auxiliary diagram is drawn, parallel to the M. R. L., and vertical lines are projected across it from the U, V, and R-points.

3.—The S-points are located on this auxiliary reference line.

4.—Begin at a point on the auxiliary reference line vertically below a known point on the M. C. L. in the first span to the left. If the end is simply supported, or if there is a known moment at the end support, as from a canti-

lever projection, the point is below this support. It may be remarked that in so far as this discussion is concerned, a cantilever projection from the end span or spans has no influence except as it determines a known point on the M. C. L. over the end support. The beam is regarded as simply supported, but subjected to a known moment at this support due to the cantilever. The cantilever itself may be considered as removed.

If the end is fixed, the point of beginning will lie vertically beneath the characteristic point adjacent to that end.

If the end is restrained, but in unknown degree, the relative fixity may be assumed, in the judgment of the designer, and a corresponding point located on the assumed M. C. L. above the support. Note, however, that it is one thing to assume a degree of fixity, and another to obtain it. Such procedure is pure guesswork, which at best can only roughly approximate the truth, and may be nowhere near it. Such assumption is only warranted when there is no rational method by which the restraint can be evaluated.

The point of beginning having been established, the J-points are then located for each span in succession, by the diagrams as indicated. It has been shown that the J-points are either on the moment closing line or vertically below the points, P, which are on that line. Therefore, each successive diagram begins at the last J-point established.

Note that to establish Points P on the M. C. L. only, it is sufficient to begin the auxiliary diagrams vertically below any known point on the M. C. L. These points, however, will not be vertically above the "fixed points", J, as here defined. Since these points are important and useful, the diagram should be begun as previously described.

5.—Draw the lines in the moment diagram connecting the U and V-points adjacent to each support. Verticals produced from the S points, intersect these lines in the required points, T.

6.—From the known point, P, on the left draw a line through the adjacent T-point to the right, for the corresponding span. The intersection of this line with a vertical produced from the corresponding J-point, determines another point, P. Beginning with this new point, the process is repeated until a point, P, is located in every span but the first, in which it must be known by condition.

7.—In the last span to the right the position of the point, P, and the end moment, or the condition of restraint, are both known, which determines the M. C. L. for that span. The intersection of this line with the vertical through the left support, is also a point on the M. C. L. for the span to the left. This point and the next point, P, to the left fix the M. C. L. for that span. The process is repeated until the line is drawn for every span.

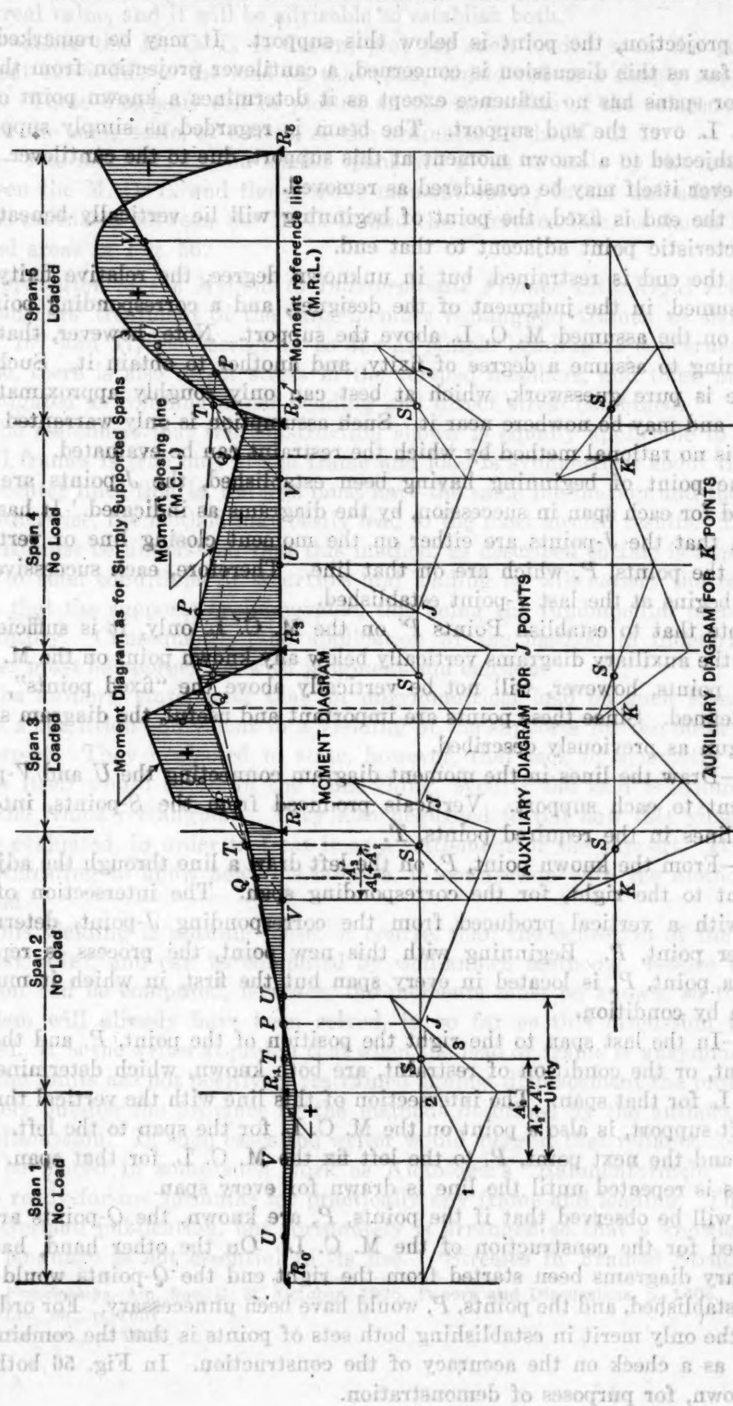

Fig. 56.

It will be observed that if the points, P, are known, the Q-points are not required for the construction of the M. C. L. On the other hand, had the auxiliary diagrams been started from the right end the Q-points would have been established, and the points, P, would have been unnecessary. For ordinary cases the only merit in establishing both sets of points is that the combination serves as a check on the accuracy of the construction. In Fig. 56 both sets are shown, for purposes of demonstration.

lever projection, the point is below this support. It may be remarked that in so far as this discussion is concerned, a cantilever projection from the end span or spans has no influence except as it determines a known point on the M. C. L. over the end support. The beam is regarded as simply supported, but subjected to a known moment at this support, due to the cantilever. The cantilever itself may be considered as removed.

If the end is fixed, the point of beginning will lie vertically beneath the characteristic point adjacent to that end.

If the end is restrained, but in unknown degree, the relative fixity may be assumed, in the judgment of the designer, and a corresponding point located on the assumed M. C. L. above the support. Note, however, that it is one thing to assume a degree of fixity, and another to obtain it. Such procedure is pure guesswork, which at best can only roughly approximate the truth, and may be nowhere near it. Such assumption is only warranted when there is no rational method by which the restraint can be evaluated.

The point of beginning having been established, the *J*-points are then located for each span in succession, by the diagrams as indicated. It has been shown that the *J*-points are either on the moment closing line or vertically below the points, *P*, which are on that line. Therefore, each successive diagram begins at the last *J*-point established.

Note that to establish Points *P′* on the M. C. L. only, it is sufficient to begin the auxiliary diagrams vertically below any known point on the M. C. L. These points, however, will not be vertically above the "fixed points", *J*, as here defined. Since these points are important and useful, the diagram should be begun as previously described.

5.—Draw the lines in the moment diagram connecting the *U* and *V*-points adjacent to each support. Verticals produced from the *S*-points, intersect these lines in the required points, *T*.

6.—From the known point, *P*, on the left draw a line through the adjacent *T*-point to the right, for the corresponding span. The intersection of this line with a vertical produced from the corresponding *J*-point, determines another point, *P*. Beginning with this new point, the process is repeated until a point, *P*, is located in every span but the first, in which it must be known by condition.

7.—In the last span to the right the position of the point, *P*, and the end moment, or the condition of restraint, are both known, which determines the M. C. L. for that span. The intersection of this line with the vertical through the left support, is also a point on the M. C. L. for the span to the left. This point and the next point, *P*, to the left fix the M. C. L. for that span. The process is repeated until the line is drawn for every span.

It will be observed that if the points, *P*, are known, the *Q*-points are not required for the construction of the M. C. L. On the other hand, had the auxiliary diagrams been started from the right end the *Q*-points would have been established, and the points, *P*, would have been unnecessary. For ordinary cases the only merit in establishing both sets of points is that the combination serves as a check on the accuracy of the construction. In Fig. 56 both sets are shown, for purposes of demonstration.

When dealing with moving loads, or influence lines, both K and J-points have real value, and it will be advisable to establish both.

8.—After the M. C. L. is drawn, plot for each loaded span the positive moment curve due to the load on that span, considered as a simply supported beam. These diagrams will be plotted on the same side of the reference line, and to the same scale, of course, as the ordinates to the M. C. L.

9.—The moments in the loaded spans are given to scale by the ordinates between the M. C. L. and the positive moment curve; in the unloaded spans, by the ordinates between the M. C. L. and the reference line, as shown by the shaded areas in Fig. 56.

Application of the Method to Indeterminate Frames with Rigid Joints.— The authors' treatment of the three-member rectangular frames, as shown in Figs. 16* and 17,† is correct in so far as simple bending is concerned. Of course, there is also direct stress in the vertical members, and these must be investigated as columns under bending and direct stress combined.

The statement that the construction shown is equally applicable to trapezoidal frames is true only if the frame and load is symmetrical about the vertical center line; that is, the legs must have the same inclination and the same EI, otherwise, the method may easily lead to the most absurd results. In general, it must be understood that this method, as described so far, is applicable only to such conditions of structure and loading as will satisfy the requirement that the supports be immovable, this being the fundamental premise of the theory. Consequently, the application of the method to indeterminate frames must be approached with judgment and caution.

The authors' statement‡ that an unsymmetrical load on such frames induces a condition equivalent to a yielding of the supports for the outer spans, is correct. They neglected to state, however, that lack of symmetry in the frame itself would result in the same thing, even if the load is symmetrical and the frame rectangular. They also neglected to tell how this yielding is to be evaluated, in order to treat it as a settlement of the support. It would be interesting to know how this is done, in connection with the methods described.

This yielding is nothing more, of course, than the deflection of the frame at that point, and can be computed by well-known methods. Before the deflection can be computed, however, the moments must be known, so that the problem will already have been solved, in so far as this discussion is concerned. It is the writer's opinion that when the load or frame is unsymmetrical, and the joints are not positively restrained against displacement the problem is entirely outside the province of the methods described by the authors, or in this discussion. In such cases the writer would suggest that reliance for short cuts be placed in some such work as Kleinlogel's "Rahmenformeln", which gives ready-for-use formulas for practically any frame and loading. The book is a German publication, but fortunately is arranged so that a knowledge of that language is not essential to its use. "Stresses in Framed Structures",

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1606.

† *Loc. cit.*, p. 1607.

‡ *Loc. cit.*, p. 1605.

previously mentioned, gives similar data, less comprehensive, but sufficient for most practical cases.

Referring to frames of the general order as shown in Fig. 18,* there might as well be several members framing into a joint as the two there shown, of course. Irrespective of the number of such members the solution is effected in the same manner, by replacing the unloaded members by one member of equal resistance to end rotation, that is, of equal rigidity.

The unloaded members at the joint create a restraining moment at the end of the loaded member, this moment being developed by the rotation of the joint through some angle, ϕ. The end of each member at the joint must necessarily turn through this same angle, and the total moment developed will be contributed by the several members in proportion to their rigidities. The measure of the rigidity of a member, in this sense, is the angle, ϕ, through which the end will turn when a certain moment, M, is applied at that end.

Consider a member of any shape, simply supported at the right end, and a moment, $M'' = 1$, applied at that end. The left end may be restrained in any degree. The moment, M' at the left end may vary from zero, if freely supported, to $\frac{x'}{L - x'}$, if fixed. (See table of notation and Fig. 53 for the significance of the symbols.) In this latter case the moment line passes through the reference line opposite the center of gravity of A'. The area, A', is based on $M' = 1$. Let the $\frac{M}{E\,I}$-area due to the moment at the left end equal A'''. Then, $A''' = A'M'$. Let ϕ equal the angle at the right end. Then, from Theorem No. 3, when $M'' = 1$, $\phi = \frac{(A'' x'' - A''' x')}{L}$.

Evidently, the rigidity, or resistance, of a member varies inversely as ϕ. Let r = this resistance, when $M'' = 1$. Then, $r = \frac{1}{\phi}$.

Let $r_1, r_2, r_3, \ldots \ldots r_n$, equal the resistances of the several unloaded members framing into a rigid joint, and let r equal the resistance of one member which would be equivalent to the group. Then, $r = r_1 + r_2 + r_3 + \ldots r_n$. Let $\phi_1, \phi_2, \phi_3, \ldots \ldots \phi_n$, equal the angles turned by the individual members when a moment, $M'' = 1$, is applied, and let ϕ equal the angle turned by the whole group at the joint when the same moment is there applied. Then, since $r = \frac{1}{\phi}$,

$$\frac{1}{\phi} = \frac{1}{\phi_1} + \frac{1}{\phi_2} + \frac{1}{\phi_3} + \ldots\ldots \frac{1}{\phi_n}$$

from which,

$$\phi = \frac{1}{\frac{1}{\phi_1} + \frac{1}{\phi_2} + \frac{1}{\phi_3} + \ldots\ldots \frac{1}{\phi_n}} \ldots\ldots\ldots\ldots (73)$$

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1608.

That is, the effect of the several members is equivalent to that of any one member of such properties that it will develop the angle, ϕ, under the application of a moment, $M'' = 1$. Since a member of any character will suffice, the simplest equivalent will naturally be adopted, this being a simply supported beam of constant $E\,I$. For this condition from Theorem No. 3,

$$\phi = \frac{A''\,x''}{L} = A'' \frac{\left(\frac{2}{3}\,L\right)}{L} = \frac{2\,A''}{3}$$

from which,

$$A'' = 1.5\,\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

For a beam of constant $E\,I$, $A'' = \frac{L}{2\,E\,I}$, and any combination of span and $E\,I$ may be used that will give the required A''; but it has been shown previously that when the point, S, is located according to the method herein described, the scale length of the spans is immaterial. Using the A'' given previously as one of the arguments in locating S, the equivalent span may be laid out to any scale. The characteristic points for this span will evidently be at the third points, since it is by condition a member of constant $E\,I$.

When a member is of constant $E\,I$, $\frac{x''}{L}$ is also a constant. Therefore, ϕ is directly proportional to A'', and if all the restraining members are of constant EI (not necessarily equal), with simply supported ends, Equation (73) may be changed to read:

$$A'' = \frac{1}{\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \ldots . \frac{1}{A_n}} \ldots\ldots\ldots\ldots (75)$$

in which, A_1, A_2, etc., are the values of A'' for the various unloaded members, and A'' is the value corresponding to the equivalent member.

If E is constant for all members, as usual, Equation (75) reduces to:

$$A'' = \frac{1}{2\,E \left\{ \frac{I_1}{L_1} + \frac{I_2}{L_2} + \frac{I_3}{L_3} + \ldots . \frac{I_n}{L_n} \right\}} \ldots\ldots\ldots (76)$$

in which, I_1, L_1, etc., refer to the properties of the individual unloaded members.

If it is desired to express the length of the equivalent member to correspond with some particular value of I (call this I_4), then Equation (76) reduces to:

$$L = \frac{I_4}{\frac{I_1}{L_1} + \frac{I_2}{L_2} + \frac{I_3}{L_3} + \ldots . \frac{I_n}{L_n}} \ldots\ldots\ldots\ldots (77)$$

which is a formula commonly used in effecting the "transformation of spans", and is equivalent to that given by the authors.*

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1606.

The moment at the joint having been found, using as an argument an equivalent member as here indicated, it is evident that this moment is to be distributed to the several component members in proportion to their values of r, or in inverse proportion to their values of ϕ, as already defined. If these members are all of constant $E\,I$ and simply supported the moments will be distributed in inverse proportion to the values of A' for the several members, or in direct proportion to their values of $\frac{I}{L}$.

To comply with the method herein described, Equation (76) is best adapted for use when all the unloaded members are of constant $E\,I$. This formula is predicated on simply supported ends for these members, but it can be used when the ends of some of or all the members are fixed, provided these members are replaced by simply supported members of equal rigidity. The simplest way of effecting this transformation is by altering the span lengths.

Let L be the length of the simply supported member, and $k\,L$ the length of the member with a fixed left end. The value of $E\,I$ is the same for both. The coefficient, k, must be such that both members will develop the same angle, ϕ'', when a moment, $M'' = 1$, is applied at the right end.

From Theorem No. 3, for the simply supported member, $\phi = \frac{2\,A''}{3}$. For the member with fixed ends, $M' = \frac{M''\,x'}{(L - x')} = 0.5$, since the inflection point is at the third point of the span. Expressing the functions for this member in terms of the length, L, the negative $\frac{M}{E\,I}$ - area $= \frac{1}{2}\,k\,A' = \frac{1}{2}\,k\,A''$, since $A' = A''$, and the positive $\frac{M}{E\,I}$ - area $= k\,A''$; but for a beam fixed at one end, $\phi = \theta$, taking the limits at the ends of the beam, and from Theorem No. 1,

$$\theta = k\,A'' - \frac{1}{2}\,k\,A'' = \frac{1}{2}\,k\,A'' = \phi''$$

This must equal the angle as found for the simply supported beam, or $\frac{k\,A''}{2} = \frac{2\,A''}{3}$, from which, $k = \frac{4}{3}$. Conversely, the member with fixed ends is the equivalent in rigidity of a member three-fourths as long with simply supported ends. Therefore, when substituting in Equation (76) the lengths of any members fixed at the ends should be inserted at three-fourths of their actual lengths.

This does not accord with the authors' statement,* by inference, that the rigidity of a member with one end fixed is $\frac{3}{2}$ that of the same member with supported ends. It is believed that that statement is in error.

Application of the Method to Settlement of Supports.—Consider a loaded span, $B\,C$, of a continuous beam of length, L. The left end is designated B. The degree of restraint at the ends is unknown.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1605.

Assume that the support, B, settles a distance, D, with respect to the support, C. Evidently, the deflection of the beam at B with respect to C, is equal to that of C with respect to B, but of opposite sign. Let M' and M'' equal the end moments as before, at B and C, respectively, exclusive of the effect of the settlement, and let M_s' and M_s'' equal the corresponding end moments induced by the settlement. For the case considered, M_s' will be positive and M_s'' negative, and the general expression for ϕ becomes:

$$\phi' = \frac{A^0\,(L - x^0)}{L} - \frac{M'\,A'\,(L - x')}{L} - \frac{M''\,A''\,(L - x'')}{L} + \frac{M_s'\,A'\,(L - x')}{L} - \frac{M_s''\,A''\,(L - x'')}{L} \ldots\ldots\ldots\ldots (78)$$

$$\phi'' = \frac{A^0\,x^0}{L} - \frac{M'\,A'\,x'}{L} - \frac{M''\,A''\,x''}{L} + \frac{M_s'\,A'\,x'}{L} - \frac{M_s''\,A''\,x''}{L} \ldots\ldots (79)$$

It has been previously proved that for beams of varying moment of inertia,

$$\frac{M''\,A''\,(L - x'')}{L} = \frac{M''\,A'\,x'}{L},$$

and,

$$\frac{M'\,A'\,x'}{L} = \frac{M'\,A''\,(L - x'')}{L}.$$

Substituting these values in Equations (78) and (79), and dividing by A' and A'', respectively, when $\phi' = \phi'' =$ zero,

$$\frac{A^0\,(L - x^0)}{A'\,L} - \left[\frac{M'\,(L - x')}{L} + \frac{M''\,x'}{L}\right] + \left[\frac{M_s'\,(L - x')}{L} - \frac{M_s''\,x'}{L}\right] = 0 \ldots\ldots\ldots\ldots (80)$$

$$\frac{A^0\,x^0}{A''\,L} - \left[\frac{M'\,(L - x'')}{L} + \frac{M''\,x''}{L}\right] + \left[\frac{M_s'\,(L - x'')}{L} - \frac{M_s''\,x''}{L}\right] = 0 \ldots (81)$$

From Equation (52) and the conditions imposed, however, the left-hand bracketed term in Equation (80) is equal to U, the height of the characteristic point, exclusive of the effect of settlement. The right-hand term in brackets, which is identical in form except for sign, is the value of a characteristic point, U', corresponding to the settlement stresses.

Similarly, from Equation (53), the left-hand bracketed term of Equation (81) is equal to V, and the right-hand term to V', the designations corresponding to those given previously. Therefore:

From Equation (80),

$$\frac{A^0\,(L - x^0)}{A'\,L} = U - U' \ldots\ldots\ldots\ldots\ldots\ldots (82)$$

From Equation (81):

$$\frac{A^0\,x^0}{A''\,L} = V - V' \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

The values of U' and V' are expressed in terms of the end moments M_s' and M_s'', which must be evaluated on the basis of the deflection, D.

Since U' and V' are characteristic points, they lie by definition on the moment closing line connecting M_s' and M''_s when the ends of the beam are

fixed, and, therefore, must be evaluated for this condition. Consider the beam as before, except that it is fixed at both ends, and there are no loads on the span. The support, B, settles with respect to C, giving rise to the positive moment, M_s', and the negative moment, M_s'', at the ends. The areas of the $\frac{M}{E I}$- diagrams are $M_s' A'$ and $- M_s'' A''$. From Theorem No. 1, the angle, θ, between the tangents at the points, B and C, is the area of the $\frac{M}{E I}$- diagram between the points, or $\theta = M_s' A' - M_s'' A''$: but $\theta = 0$ by condition; therefore, $M_s' A' = M_s'' A''$ from which, $M_s'' = \frac{M_s' A'}{A''}$.

Regarding the deflection of B with respect to C, the beam may be considered as a cantilever fixed at C, and subjected to a combination of downward pull and positive moment at B, resulting in the deflection, D. The pull at B generates a moment, $-M_s''$, at C, and the moment applied at B is M_s'. The corresponding $\frac{M}{E I}$- areas are $- M_s'' A''$ and $M_s' A'$, the distances of their centers of gravity from B being x'' and x', respectively. Then, from Theorem No. 4, the value of the deflection will be:

$$D_B = - D_C = \phi'' L + M_s' A' x' - M_s'' A'' x''$$

but $\phi'' =$ zero by condition, and it has been proved that $M_s'' = \frac{M_s' A'}{A''}$.

Therefore,

$$D_B = - D_C = M_s' A' x' - M_s' A' x'' = M_s' A' (x' - x''),$$

from which,

$$M_s' = \frac{D}{A' (x' - x'')}$$

Similarly,

$$M'' = \frac{- D}{A'' (x' - x'')}.$$

From Equation (52):

$$U' = \frac{D (L - x')}{L A' (x' - x'')} - \frac{D x'}{L A' (x' - x'')} \qquad (84)$$

From Equation (53):

$$V' = \frac{D (L - x'')}{L A' (x' - x'')} - \frac{D x''}{L A'' (x' - x'')} \qquad (85)$$

From Equation (82):

$$U = \frac{A^0 (L - x^0)}{A' L} + \frac{D (L - x')}{A' L (x' - x'')} - \frac{D x'}{A'' L (x' - x'')} \qquad (86)$$

From Equation (83):

$$V = \frac{A^0 x^0}{A'' L} + \frac{D (L - x'')}{L A' (x' - x'')} - \frac{D x''}{L A'' (x' - x'')} \qquad (87)$$

Equations (86) and (87) give the heights of the characteristic points for the combined condition of load and settlement. The second and third terms on the right-hand side are the heights of characteristic points as for the settlement alone; the first term on the right-hand side is the normal height, without settlement.

When the member is symmetrical about the center of the span, so that $A' = A''$, and $x'' = L - x'$, Equation (84) reduces to $\frac{-D}{A' L}$, and Equation (86) to:

$$U = \frac{A^0 (L - x^0)}{A' L} - \frac{D}{A' L} \dots\dots\dots\dots\dots\dots (88)$$

Equation (85) reduces to $\frac{D}{A' L}$, and Equation (87) becomes:

$$V = \frac{A^0 x^0}{A' L} + \frac{D}{A' L} \dots\dots\dots\dots\dots\dots (89)$$

When $E\,I$ is constant throughout the beam, $A' = A'' = \frac{L}{2\,E\,I}$, and Equations (88) and (89) reduce to:

$$U = \frac{A (L - x^0)}{L^2} - \frac{2\,E\,I\,D}{L^2} \dots\dots\dots\dots\dots\dots (90)$$

$$V = \frac{A\,x^0}{L^2} + \frac{2\,E\,I\,D}{L^2} \dots\dots\dots\dots\dots\dots (91)$$

In general, the correctness to the normal heights of the characteristic points are subtractive for the two points immediately adjacent to the settled support, and additive to the other two. The settlement of a support effects the characteristic points only in the two spans common to the support.

Conclusion.—As previously stated, the writer lays no claim to originality as respects the fundamental conceptions on which this discussion is based. The entire credit for these belongs primarily to Fidler, and, secondarily, to Ostenfeld and others who have added in one respect or another to the structure for which Fidler laid the foundation. The writer accepts entire responsibility for the particular manner in which the concept is herein developed and expanded, and trusts that he has succeeded in adding something to the existing literature on the subject. The purpose throughout has been to develop and present formulas for the purely general case, and to lay stress on the true significance of these formulas and the graphical operations involved, rather than to deal with specific instances of limited application. This has necessarily resulted in some rather intricate and involved operations, which may severely test the forbearance of the reader. If the writer has failed to demonstrate the inherent simplicity and excellence of Fidler's Method of Characteristic Points, this is due only to the limitations of his ability to present the subject.

The writer wishes to express his appreciation of the assistance rendered by Mr. Ruppel and H. A. Schirmer, Jun. Am. Soc. C. E., in having given this manuscript a critical and careful reading, and in making suggestions which have helped to determine the final form in which the discussion is presented.

A. T. GRANGER,* Assoc. M. Am. Soc. C. E. (by letter).†—Two distinct subjects are presented in this paper—the simplified form of the Theorem of Three Moments, and the graphical solution of the theorem by the Method of Conjugate Points.

The writer has been particularly interested in the simplified form and derivation of the three-moment equation. Various methods of deducing this equation have been published, but this derivation appears simpler and more easily understood than any previously proposed. The form given in Equation (7)‡ can easily be converted to the familiar form by substituting for the right-hand members, $g_1 A_1$ and $g_2 A_2$, their values in terms of the loads, positions, and span lengths expressed in the ordinary notation.

The graphical solution of the equation by the Method of Conjugate Points possesses two distinct advantages over the analytical method, namely, the check on the correctness of the construction in Fig. 11,§ and the simplicity of the graphical method when applied to several spans.

A positive check on the correctness of the moments over the supports is advantageous in eliminating errors that would otherwise be carried through the succeeding calculations. Any one familiar with computations for continuous girders is aware that an error in these moments cannot be detected by any failure of the reactions, shears, or moments based thereon to check. The writer has frequently examined calculations which would satisfy all the laws of statics, but which were wrong throughout because of an error in solving the three-moment equation. Such an error is much less likely to occur when the moments are determined graphically.

Perhaps the principal advantage of the graphical method lies in its easy extension to cases involving more than two unknown moments. Although the actual solution of such cases in practice is comparatively infrequent, there are doubtless numerous instances where a more complete analysis would be made were it not for the labor involved. The complication of the analytical method increases rapidly with each additional unknown moment. The graphical method, on the contrary, is easily extensible to any number of spans and such extension offers no additional difficulty. This characteristic should commend it for use in analyzing complex structures.

Study of this paper led the writer to the conclusion that an alternative graphical solution utilizing the familiar equilibrium polygon would possess some advantages. He accordingly devised the following method of procedure, which is presented with the hope that it may prove interesting and of some value.

Fig. 57(*a*) represents a continuous beam loaded in any fashion. Although a three-span girder has been chosen for illustration, it will be obvious that the method developed is general and applicable to any number of spans. The simple beam moment diagrams, based on discontinuity, are plotted in Fig. 57(*b*), the moment areas, A_1, A_2, and A_3, determined, and the centers of

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† Received by the Secretary, January 8, 1926.

‡ *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1602.

§ *Loc. cit.*, p. 1600.

gravity of these areas located, vertical lines being drawn through such centers of gravity. The areas A_1, A_2, and A_3, are now laid off in Fig. 57(*c*), as vertical forces to any convenient scale, a pole, *O*, chosen at a pole distance, *H*, and the rays drawn as shown. In Fig. 57(*d*), an equilibrium or string polygon, *L m n q Q*, is now constructed using these rays. If, instead of representing A_1, A_2, and A_3, as single forces in Fig. 57(*c*), these areas had been divided into a great number of minute areas, the corresponding equilibrium polygon in

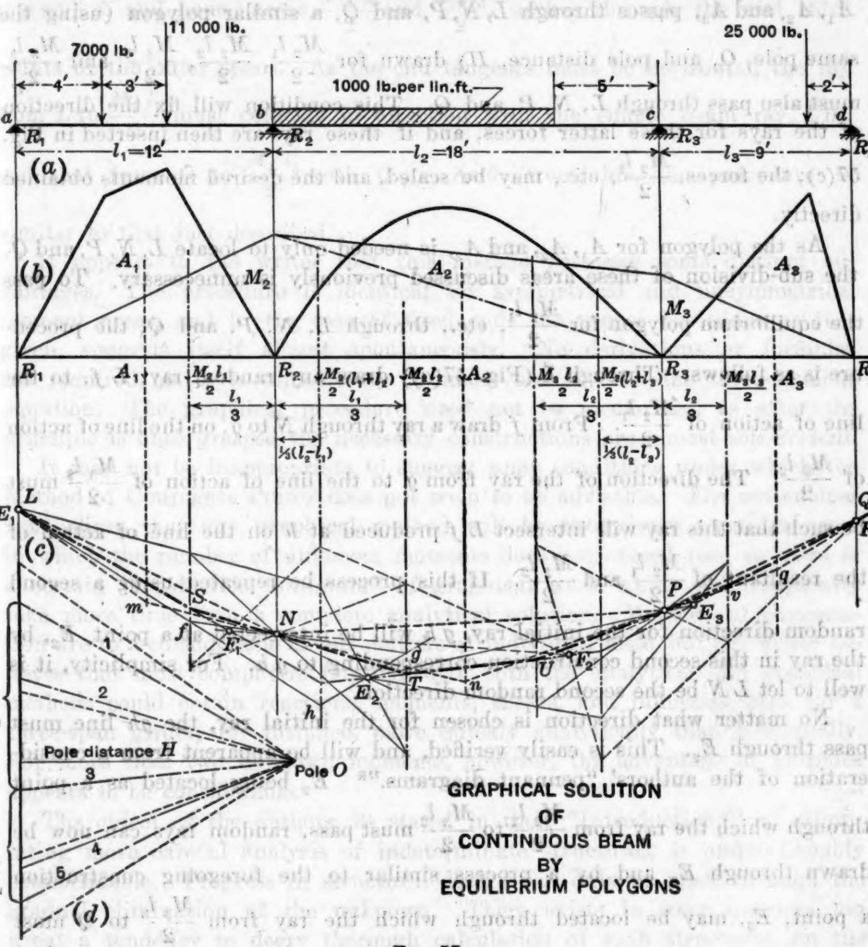

FIG. 57.

Fig. 57(*d*), instead of the broken line, *L m n q Q*, would have been practically a smooth curve. This curve represents the elastic curve of the beam (considered as three simple spans);* and straight lines connecting the points, *L*, *N*, *P*, and *Q*, where the polygon intersects the reaction verticals, will represent the unstrained position of the beam.

* "Modern Framed Structures", by Messrs. Johnson, Bryan and Turneaure, Pt. II, 1917 Edition, pp. 5-6.

The correct elastic curve for the continuous beam may be obtained by combining with this simple beam curve a similar curve representing the effect of the areas due to M_2 and M_3, the moments over the supports due to continuity. (See Fig. 57(*b*)). The net displacement of the beam at points *a, b, c,* and *d* (Fig. 57(*a*)), is obviously zero; and as it is zero due to the effect of the areas A_1, A_2, and A_3, alone, it must be zero due to the areas caused by M_2 and M_3 alone. In other words, inasmuch as the equilibrium polygon for A_1, A_2, and A_3, passes through *L, N, P,* and *Q,* a similar polygon (using the same pole, *O*, and pole distance, *H*) drawn for $\frac{M_2\, l_1}{2}$, $\frac{M_2\, l_2}{2}$, $\frac{M_3\, l_2}{2}$, and $\frac{M_3\, l_3}{2}$, must also pass through *L, N, P,* and *Q*. This condition will fix the direction of the rays for these latter forces, and if these rays are then inserted in Fig. 57(c), the forces, $\frac{M_2\, l_1}{2}$, etc., may be scaled, and the desired moments obtained directly.

As the polygon for A_1, A_2, and A_3, is needed only to locate *L, N, P,* and *Q,* the sub-division of these areas discussed previously is unnecessary. To pass the equilibrium polygon for $\frac{M_2\, l_1}{2}$, etc., through *L, N, P,* and *Q*, the procedure is as follows: Through *L* (Fig. 57(*a*)) draw any random ray, *L f*, to the line of action of $\frac{M_2\, l_1}{2}$. From *f* draw a ray through *N* to *g*, on the line of action of $\frac{M_2\, l_2}{2}$. The direction of the ray from *g* to the line of action of $\frac{M_3\, l_2}{2}$ must be such that this ray will intersect *L f* produced at *h* on the line of action of the resultant of $\frac{M_2\, l_1}{2}$ and $\frac{M_2\, l_2}{2}$. If this process be repeated using a second random direction for the initial ray, *g h* will be intersected at a point, E_2, by the ray in this second construction corresponding to *g h*. For simplicity, it is well to let *L N* be the second random direction.

No matter what direction is chosen for the initial ray, the *gh* line must pass through E_2. This is easily verified, and will be apparent from a consideration of the authors' "pennant diagrams."* E_2 being located as a point through which the ray from $\frac{M_2\, l_2}{2}$ to $\frac{M_3\, l_2}{2}$ must pass, random rays can now be drawn through E_2 and by a process similar to the foregoing construction a point, E_3, may be located through which the ray from $\frac{M_3\, l_3}{2}$ to *Q* must pass. The points, E_2 and E_3, suffice to locate the required polygon, but as a check the points, F_2 and F_1, should be similarly located by starting at *Q* and working in the reverse direction. The complete polygon, *L S T U V Q,* is shown in Fig. 57(*d*).

In Fig. 57(*c*), Rays 1, 2, 3, 4, and 5 are now drawn parallel, respectively, to *L S, S T, T U, U V,* and *V Q*. It is at once apparent that Rays 2 and 4 are

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, pp. 1597–1598.

not essential and that the intercept on the load line between Rays 1 and 3 is $M_2\left(\frac{l_1 + l_2}{2}\right)$, while that for Rays 3 and 5 is $M_3\left(\frac{l_2 + l_3}{2}\right)$. From these intercepts the values of M_2 and M_3 are obtained directly.

In case the ends are fixed there will be moments, M_1 and M_4, existing at a and d, and hence areas, $\frac{M_1\, l_1}{2}$ and $\frac{M_4\, l_3}{2}$, acting through the end third points of the outer spans. As the end tangents must be horizontal, the ray from L to $\frac{M_1\, l_1}{2}$ must coincide in direction with the simple beam ray, $L\,m$, and that from Q to $\frac{M_4\, l_3}{2}$ with $Q\,q$. Aside from this, the procedure is similar to that just described.

It appears to the writer that this method possesses some distinct advantages. The procedure is identical for symmetrical and unsymmetrical moment areas, and in the case of fixed ends the proper procedure as just given, suggests itself almost spontaneously. No derivations or formulas are required, as it is unnecessary to make use of even the three-moment equation. The graphical procedure need not be memorized, as after the principle is once grasped the necessary constructions are almost self-evident.

It may not be inappropriate to suggest some conditions under which the Method of Conjugate Points does not seem to be advisable. The writer does not believe that any graphical method will be extensively used for cases in which the number of unknown moments does not exceed two, as there is a certain "irreducible minimum" of graphical work which will frequently take more time than a complete analytical solution. Methods of computation are to a considerable extent matters of individual taste, but the writer believes that most computers familiar with both the analytical and graphical methods could obtain reactions, moments, shears, and influence lines for a three-span girder, for instance, more quickly analytically than graphically. For more than two unknown moments, however, the advantage of graphics appears to be considerable.

The object of the authors, as stated in their "Introduction,"* of stimulating more careful analysis of indeterminate structures, is unquestionably commendable. Progress in structural design can only be made through the gradual elimination of the unknown. There exists in some quarters too great a tendency to decry thorough calculation of such structures, on the ground that the entire subject is enveloped in such a haze of uncertainty as to defy analysis. It will not do to lose sight of the fact that those who are best acquainted with the so-called "exact" methods of calculation should be best qualified to decide what short cuts are permissible, and to what degree of refinement the analysis need be carried. For calling attention to the importance of this subject, and for their valuable addition to its literature, the authors should receive the thanks of the profession.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1592.

THE HEXAGONAL SLAB DESIGN OF CONCRETE PAVEMENT

Discussion*

By Messrs. S. Herbert Hare and Jacob Feld

S. Herbert Hare,† Esq. (by letter).‡—The writer has had the opportunity to watch the development of this type of slab under Mr. Perry's supervision in street paving at Longview, Wash., and has been much interested in the results. His interest, however, is not primarily in the strength, economy, or efficiency of this type of slab, which points are so ably discussed in the paper and which are so vital to the engineer, but in the appearance of the paving as laid on the streets. Any longitudinal joint, whether continuous or broken, appears very conspicuous in the perspective view as one drives down the street. The hexagonal joints, having no sides parallel to the curb, soon lose themselves from view. In most cases they cannot be seen farther than 150 to 200 ft. away. Beyond that the pavement appears as one unbroken slab.

Although often secondary from the point of view of the engineer, the esthetic effect of any paving is quite important to those who use the street, especially when such paving is used on boulevards and parked roads, as in Longview.

Jacob Feld,§ Assoc. M. Am. Soc. C. E. (by letter).‖—The author plainly states his thesis and proves it. The right-angled corner is a pronounced weakness in a pavement. Both experimentally and theoretically, he shows why right angles should find no rightful place in a pavement design. Then, as a solution to the problem, he recommends the use of hexagonal units, noting as special advantages the decreased danger from temperature changes and the elimination of the right-angled corners. However, as is plainly seen in Fig. 6,¶ the proposed type does not eliminate the right-angled corners along the outer edges of the pavement. It is along these edges that the greatest stresses occur and to allow for these, Mr. Perry recommends greater thickness of the slab along the edges—the same remedy as is used in the present type of construction.

There is no special advantage in having the intersections of the joints with the edges staggered; nor is the elimination of the square joints from the

* Discussion on the paper by Lewis A. Perry, Assoc. M. Am. Soc. C. E., continued from January, 1926, *Proceedings*.

† Kansas City, Mo.

‡ Received by the Secretary, January 7, 1926.

§ Cons. Engr., New York, N. Y.

‖ Received by the Secretary, January 19, 1926.

¶ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1801.

center line of great importance. The advantage gained from the use of smaller units can be just as easily obtained by increasing the number of expansion joints in the square arrangement. The elimination of the straight center joint takes away the now common traffic line, separating the streams of travel. There is no doubt that hexagonal slabs are of special value in paving large areas, as shown in Fig. 5,* and should be used in such cases, especially since their cost is no more than that of square slabs.

A useful application of the hexagonal idea for soil bearing is for column footings. A hexagonal footing contains the same volume of concrete as an equivalent square footing, with somewhat less steel reinforcement and a smaller area of forms. The cost is practically the same. Hexagonal footings are used only where space limitations govern, or for extremely large loads as, for example, high stacks.

The author has brought out a novel idea and has proven its advantages; but the writer cannot see where its use can be justified in the ordinary type of pavement.

* *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1801.

STREAM POLLUTION

A SYMPOSIUM

Discussion*

By Kenneth Allen, M. Am. Soc. C E.

Kenneth Allen,† M. Am. Soc. C. E. (by letter).‡—The four papers§ by Messrs. Frost, Theriault, Streeter, and Hoskins present an admirable summary of the notable investigations carried on by the U. S. Public Health Service on stream pollution which constitute a distinct advance in studies of this subject.

The pollution of bodies of salt water presents similar problems and although the principles controlling oxidation and reaeration are the same, the rates at which these take place and the products of decomposition are, because of the salinity of the water, quite different. Moreover, the tides present conditions as to velocity, direction of flow, and degree of dilution, that are quite at variance with those found in inland waters.

These modifying conditions as related to the problem of New York Harbor are thoroughly discussed in the three reports—issued in 1910, 1912, and 1914—of the Metropolitan Sewerage Commission and in the excellent paper‖ by H. de B. Parsons, M. Am. Soc. C. E. This discussion is confined to a résumé of the studies and work looking to a better control over the pollution of the waters of New York Harbor that have been carried out since 1914.

Dissolved Oxygen.—Chemical examinations to detect pollution have been practically confined to tests for dissolved oxygen,¶ as this has been found the best criterion of pollution for which field tests can be readily made. Beginning with the results obtained by the Metropolitan Sewerage Commission in 1909 there is, with the exception of 1910, a continuous record of warm-weather saturations up to the present time.**

The average and minimum percentages found at certain representative stations and the average for the main branches of the harbor in 1909 and 1924 are given in Tables 18 and 19. About one-half the samples were taken from near the surface and one-half from near the bottom so that the averages furnish a fair measure of the general condition of the waters.

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* Continued from February, 1926, *Proceedings*.

† San. Engr., Office of Chf. Engr., Board of Estimate and Apportionment, New York, N. Y.

‡ Received by the Secretary, November 24, 1925.

§ *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, pp. 1809–1855.

‖ "Tidal Phenomena in the Harbor of New York," *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 1979.

¶ A limited number of oxygen demand, oxygen consumed, and hydrogen-ion tests have also been made in connection with special studies.

** Published in 1917 and subsequently in the Annual Report of the Chief Engineer of the Board of Estimate and Apportionment, New York, N. Y.

TABLE 18.—SATURATIONS OF DISSOLVED OXYGEN AT SELECTED STATIONS, 1909 AND 1924.

STATION.	SATURATION. Minimum percentage and date. 1909.	SATURATION. Minimum percentage and date. 1924.	SATURATION. Number of samples and average percentage. 1909.	SATURATION. Number of samples and average percentage. 1924.
EAST RIVER:				
Throgs Neck	88%-June 23	39%-July 17	25- 96%	14-56%
42d Street	57% [illegible]	14%-Aug. 6	4- 63%	30-23%
23d Street	52%-July 2	15%-{Aug. 6, " 21}	8- 62%	30-24%
Pier 10	43%-June 25	12%-July 10	20- 62%	32-31%
HUDSON RIVER:				
Mt. St. Vincent	60%-July 7	30%-July 8	4- 70%	14-59%
Spuyten Duyvil	55%-July 7	26%-July 16	4- 71%	14-53%
155th Street	69%-July 26	27%-Sept. 2	1- 69%	14-47%
42d Street	65%-Sept. 7	22%-Aug. 7	5- 74%	18-40%
Pier A	57%-July 22	25%-July 8	11- 67%	38-41%
HARLEM RIVER:				
Morris Heights	46%-July 15 (University Heights)	10%-Sept. 9	4- 53% (University Heights)	16-39%
Willis Avenue	32%-July 15 (Third Avenue)	0%-{July 8, Aug. 6}	16- 56% (Third Avenue)	16-11%
106th Street	21%-July 15 (109th Street)	1%-{July 30, Aug. 21}	6- 41% (110th Street)	16-14%
UPPER BAY:				
Bell Buoy 26	60%-July 16	28%-Aug. 13	14- 73%	48-42%
Robbins Reef	62%-July 22	31%-June 26	3- 70%	111-52%
The Narrows	62%-June 24	42%-July 10	16- 78%	16-73%
KILL VAN KULL				
Shooters Island	78%-Sept. 16	85%-July 2	2- 78%	16-46%
ARTHUR KILL:				
Opposite Fresh Kills	71%-Sept. 16	84%-Aug. 28	4- 85%	16-71%
Tottenville Ferry	100%-Sept. 21	61%-Aug. 28	2-101%	12-88%
JAMAICA BAY:				
Barren Island	78%-Sept. 17	81%-Aug. 15	2- 80%	8-93%
Beach Channel and Long Island Railroad	80%-June 29	84%-July 25	2- 80%	8-96%
Bergen Beach	67%-Sept. 17	63%-Sept. 12	2- 68%	1-63%
Mouth of Paerdergat Creek	....	20%-July 1	....	5-49%

The general average summer saturations in the main branches of the harbor are shown in Table 20.

The most striking loss of dissolved oxygen is seen to be in the Lower East River, where the degree of saturation in 1924 was only 40% of that of 1909.

In the summer of 1921 exceptionally low saturations were noted:

At Throgs Neck, an average of 49 and a minimum of 24 per cent.

At the Narrows, an average of 35 and a minimum of 15 per cent.

In the Hudson River, an average of 30 and a minimum of 12 per cent.

In the Lower East River, an average of 16 and a minimum of 0 per cent.*

In the Harlem River, an average of 15 and a minimum of 0 per cent.

Remembering that a few days of complete depletion in warm weather would result in a very serious condition it is significant that from past experience

* Off 42d Street, September 13, 1921.

minimum percentages may be expected any summer, not exceeding about 20% in the Hudson River and 10% in the East River, whereas in the Harlem River complete depletion is noted at times every summer.

TABLE 19.—DISSOLVED OXYGEN—AVERAGES OF SAMPLES ANALYZED BETWEEN JUNE 1 AND OCTOBER 1 FOR EACH OF THE MAIN BRANCHES OF NEW YORK HARBOR.

Year.	Total number of samples.	AVERAGE PERCENTAGE OF SATURATION.						
		Hudson River below Spuyten Duyvil.	Harlem River.	Upper East River.	Lower East River.	Upper Bay.	Kill van Kull.	The Narrows.
1909*	404	72	55	86	65	67	79	83
1911*	861	62	42	69	54	72	70	76
1912*	150	58	..	..	49	64	65	71
1913*	880	57	29	..	43	66	65	69
1914	473	50	30	50	40	71	..	68
1915	245	43	28	..	33	72	..	78
1916	176	46	24	..	26	64	..	63
1917	238	42	22	47	29	50	..	63
1918	54	54	23	50	21	56	..	61
1919	820	36	29	30	24	51	35	58
1920	264	44	23	50	27	43	42	52
1921	258	30	15	37	16	33	38	35
1922	280	44	26	51	26	51	51	60
1923	354	37	27	38	22	47	43	57
1924	627	44	26	45	26	48	48	73

* By Metropolitan Sewerage Commission.

TABLE 20.—DISSOLVED OXYGEN—AVERAGE PERCENTAGE OF SATURATIONS IN MAIN BRANCHES OF NEW YORK HARBOR, JUNE 1 TO OCTOBER 1.

Locality.	1909.	1914.	1924.	Percentage of 1909 value.
The Narrows	78	68	75	94
Upper Bay, Robbins Reef	70	71	52	74
Hudson River	72	50	44	61
Harlem River	55	30	26	47
Upper East River	86	50	45	52
Lower East River	65	40	26	40

Population and Sewage Flow.—The chief cause of harbor pollution is the sewage from the population of the Metropolitan District,* now estimated at 8 700 000.

The population of New York City was 5 620 048, or, excluding sailors on ships and those living on islands in the East River, about 5 586 500, in 1920, and it is estimated that by 1960 this figure will have increased to 11 900 000, as indicated in Table 21.

* Including White Plains, N. Y., on the north, the mouth of Raritan River on the south, the New York City limits on the east, and Paterson, Summit, and Perth Amboy, N. J., on the west; in all about 700 sq. miles, within a range of from 15 to 20 miles from New York City Hall, Report, Metropolitan Sewage Comm., 1910.

TABLE 21.—POPULATION OF NEW YORK, N. Y., RECORDED IN 1920 AND EXPECTED IN 1960.

Tributary in 1920 to	Manhattan.	The Bronx.	Queens.	Brooklyn.	Richmond.	Total.
Hudson River	793 000	2 000	....	....	....	795 000
Harlem River	607 000	406 000	....	....	....	1 013 000
Upper East River	....	323 000	127 000	....	....	450 000
Lower East River	857 000	....	186 000	942 000	....	1 985 000
Upper Bay	....	....	....	696 000	33 000	729 000
Kill van Kull	....	....	....	....	48 500	48 500
Arthur Kill	....	....	....	....	18 000	18 000
Jamaica Bay	....	....	156 000	373 000	....	529 000
Lower Bay	....	....	....	....	17 000	17 000
Long Island Sound	....	2 000	....	....	....	2 000
Total	‡2 257 000	*733 000	*469 000	†2 011 000	116 500	5 586 500
Tributary in 1960 to						
Hudson River	1 499 000	511 000	....	....	....	2 010 000§
Harlem River	421 000§	631 000§	....	....	....	1 052 000
Upper East River	....	962 000	850 000	....	....	1 812 000
Lower East River	869 000	....	401 000	1 094 000	....	2 364 000
Upper Bay	....	....	....	1 523 000	153 500	1 676 500
Kill van Kull	....	....	....	....	408 500	408 500
Arthur Kill	....	....	....	....	280 000	280 000
Jamaica Bay	....	....	849 000	1 183 000	....	2 032 000
Lower Bay	....	....	....	....	258 000	258 000
Long Island Sound	....	7 000	....	....	....	7 000
Total	2 789 000*	2 111 000*	2 100 000	3 800 000	1 100 000	11 900 000

* Marble Hill (population 1 000 in 1920; 11 000 in 1960) included for sewerage studies in The Bronx.

† Sailors on ships, 7 000.

‡ Population on outlying lands and sailors on ships, 26 000.

§ Includes population on some areas at present draining to Harlem River, but intended eventually for the Hudson River (295 000 from Manhattan and 292 000 from Jerome Avenue area, The Bronx).

In the entire Metropolitan District, as defined by the Metropolitan Sewerage Commission,* there were 5 834 857 inhabitants in New York State and 1 863 577 in New Jersey—a total of 7 698 434—in 1920, and by 1960 it is believed these figures will be increased to 12 312 000 in New York and 3 377 000 in New Jersey, or a total of 15 689 000. The distribution of these populations is given by counties in Table 22.

The volume of sewage entering New York Harbor from New York City is estimated at 940 000 000 gal. per day for 1920 and 1 757 000 000 gal. per day for 1960, and the quantity from each Borough to each of the main divisions of the harbor is given in Tables 23 and 24. From these tables it will be seen that about four-fifths of the total quantity is received above The Narrows, through which it flows to the Lower Bay. These estimates are based on the water supply, which averaged 131 gal. per capita of the residential population in 1920 and is estimated to average 130 gal. per capita in 1960, to which is added a small volume due to infiltration of ground-water.

Aside from the sewage of New York City there is that from the outlying population. If this is taken as averaging 80 gal. per capita daily, it amounts

* Including White Plains, N. Y., on the north, the mouth of Raritan River on the south, the New York City limits on the east, and Paterson, Summit, and Perth Amboy, N. J., on the west; in all about 700 sq. miles, within a range of from 15 to 20 miles from New York City Hall, Report, Metropolitan Sewage Comm., 1910.

to 166 000 000 gal. per day from a population of 2 078 386 in 1920 and will amount to 303 000 000 gal. per day from 3 788 996 persons in 1960. The total volume then produced in the Metropolitan District will be 2 060 000 000 gal. per day.

TABLE 22.—POPULATION OF METROPOLITAN DISTRICT, RECORDED IN 1920 AND EXPECTED IN 1960.

	1920.	1960.
NEW YORK:		
New York City	5 620 048	11 900 000
Westchester County	212 172	405 924
Nassau County	2 637	6 053
Total	5 834 857	12 311 977
NEW JERSEY:		
Bergen County	148 741	365 953
Essex County	634 946	1 152 794
Hudson County	629 154	1 037 042
Middlesex County	55 130	111 384
Passaic County	239 217	375 021
Union County	156 389	334 825
Total	1 863 577	3 377 019
Total in Metropolitan District	7 698 434	15 688 996

TABLE 23.—SEWAGE DISCHARGED INTO NEW YORK HARBOR FROM THE CITY OF NEW YORK IN 1920.

Source.	Million Gallons Per Day.					
	Manhattan.	The Bronx.	Brooklyn.	Queens.	Richmond.	Total.
Hudson River	159	1	....	....	....	160
Harlem River	68	72	....	....	....	140
Upper East River	....	65	....	46	....	111
Lower East River	137	....	103*	27*	....	267*
Upper Bay	....	....	88	....	9	97
Kill van Kull	....	....	....	....	15	15
Total above The Narrows						790
Lower Bay	....	....	....	....	17	17
Jamaica Bay	....	....	59	52	....	111
Arthur Kill	....	....	....	....	22	22
Total below The Narrows						150
Grand total						940

* Allowing for 14 000 000 gal. per day diverted from the Borough of Queens to the East River at North 12th and South 5th Streets, Brooklyn.

Causes of Pollution.—As already mentioned, the chief cause of harbor pollution is the sewage from the 8 700 000 inhabitants of the Metropolitan Dis-

trict. That entering each main branch of the harbor from each Borough of New York in 1920 and as forecast for 1960 is given in Table 23. With a uniform increase the volume in 1925 may be taken at 1 050 000 000 gal. per day, and assuming a daily volume of 80 gal. per capita from the 2 300 000 persons in the outlying sections of the Metropolitan area the total volume now produced, in gallons per day, is,

$$1\,050\,000\,000 + (80 \times 2\,300\,000 = 184\,000\,000) = 1\,234\,000\,000$$

TABLE 24.—SEWAGE DISCHARGED INTO NEW YORK HARBOR FROM THE CITY OF NEW YORK IN 1960.

Source.	Million Gallons Per Day.					
	Manhattan.	The Bronx.	Brooklyn.	Queens.	Richmond.	Total.
Hudson River	248.08*	76.83†	......	......	......	324.91
Harlem River*	......	......	......	......	......	......
Long Island Sound	......	1.27	......	......	......	1.27
Upper East River	......	150.14	......	104.85¶	......	254.99
Lower East River	212.14§	92.83‖	160.04‡	55.07‡¶	......	520.08
Upper Bay	......	......	194.08	......	21.16	215.24
Kill van Kull	......	......	......	......	54.49	54.49
Total above The Narrows						1370.98
Lower Bay	......	......	......	......	43.48	43.48
Jamaica Bay	......	......	158.45	133.40	......	291.85
Arthur Kill	......	......	......	......	51.09	51.09
Total below The Narrows						386.42
Grand total						1757.40

*Assuming 48 750 000 gal. per day diverted from the Harlem River to the Hudson River.
†Assuming 43 260 000 gal. per day diverted from the Harlem River to the Hudson River.
‡Assuming 13 970 000 gal. per day diverted from Queens to Brooklyn.
§Assuming 69 000 000 gal. per day diverted from the Harlem River to the Lower East River.
‖Assuming 92 830 000 gal. per day diverted from the Harlem River to the Lower East River.
¶Assuming 25 170 000 gal. per day diverted from the Upper East River to the Lower East River.

Of the sewage received from outside of New York City by far the most important contribution is from the Passaic Valley Sewer District. The population of this District is estimated at 995 000 and the sewage delivered to the outlet is understood to approximate 135 000 000 gal. per day. No such volume is received at any other outlet in the Metropolitan District, the 1 050 000 000 gal. per day from the City of New York being discharged through about 465 outlets, distributed as follows:

Manhattan	190
The Bronx	30
Brooklyn	100
Queens	95
Richmond	50

Besides the domestic sewage there is more or less industrial waste. This is an important consideration in the Passaic Valley Sewer District where there are large woolen, silk, dye, chemical, and metal works and where their waste constitute a material part of the total flow. In New York there are large gas and oil works and a limited number of other plants, such as packing houses, dye works, etc., while along the New Jersey shores of the Arthur Kill there are important chemical, copper, oil, asphalt, and fertilizer works. Within the New York area industrial wastes, aside from oil, do not materially affect general conditions. Although it is admitted that, in spite of the required use of separators at garages, large quantities of gasoline and case oil reach the sewers and are the cause of occasional explosions, the total domestic sewage is so much larger in proportion that oil from these sources is not recognizable at most sewer outlets. The fields of sleek and oil found on the open water are mainly due to the surreptitious pumping out of oil tanks on vessels, from careless or accidental spills at refineries and gas works, and from the wastes received or washed off by the tide at ship-repair yards.

Sludge beds increasing the depletion of oxygen in the outlying waters are prevalent in the slips, in Wallabout Basin, Gowanus Canal, Newtown Creek, Newark Bay, off Hoboken, Bergen Point, and Elizabethport. The channels of the Lower East River and the Jersey Flats are relatively free from deposits.

Disregarding the depletion of dissolved oxygen due to sludge deposits or other than domestic sewage and assuming 0.22 lb. per capita, the writer roughly estimated the biological oxygen demand of the harbor waters in 1920 at 1 558 700 lb. per day.

To offset the effect of this demand on the oxygen of the harbor there is the replenishment of oxygen due to that brought in by the tides and fresh-water streams, that absorbed from the air, and a certain amount set free by aquatic vegetation. In some streams the latter may be the controlling factor, and at times this is probably true of Jamaica Bay, where samples frequently show supersaturation. In New York Harbor, however, as a whole, one would not be warranted in relying on so uncertain and temporary a supply. Disregarding the production of oxygen by algæ and other vegetation, and making certain rather crude assumptions as to average saturations, salinity, temperature, and stream flow, the oxygen supplied by incoming tidal and upland waters was estimated* at 1 000 600 lb., and that due to surface absorption at 863 100 lb. per day, leaving an excess available for purposes of purification of 305 000 lb. per day, provided diffusion were complete and no other factors were involved. Where the several basic conditions are in a state of such constant flux and where so many assumptions must be made on incomplete supporting data, estimates of this kind must be taken with reservation and considered merely as indicating possible conditions.

The importance of recognizing the effect of reaeration in any forecast of conditions is made very clear by Mr. Streeter† and from his able analysis of the results secured in studying the Ohio River such forecasts may be made

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 718.

† *Proceedings*, Am. Soc. C. E., November, 1925, Papers and Discussions, p. 1829.

in the case of fresh-water streams of known characteristics with reasonable confidence. The reaeration of quiescent sea water is probably at a slower rate than that of fresh water although diffusion is as a rule more rapid. When disturbed by winds, eddies, or vessels, the rate of absorption is increased by bringing the less highly oxygenated water to the surface. Owing to the limited depth affected by these causes the increased rate is less in deep than in shallow waters.

These influences are fully discussed by W. M. Black, M. Am. Soc. C. E., and Earle B. Phelps, Affiliate, Am. Soc. C. E., in Chapter II of their report to the Chief Engineer of the Board of Estimate and Apportionment of New York City dated February 16, 1911. In this report the attempt is made to find an average rate of absorption for the waters of the harbor after making due allowance for salinity, depth, temperature, and the mixing action due to tides, vessels, and wind. They say:

"The action of wind and tide reverses the conditions found in quiescent water, in that the rate of absorption of oxygen per million gallons increases slightly with increasing depth down to 30 ft. With the added action of boats this effect is not so noticeable, and in the final results the combined effect results in a very even rate of absorption so that special treatment of the 12-ft. depths need no longer be considered and for all practical purposes the average rate of absorption in the Bay and Lower Hudson may be taken at 0.045, in the East River at 0.050, and in the Upper Hudson at 0.019 lb. per hour per million gallons of water in all depths less than 40 ft. The effect of aeration in depths greater than 40 ft. may safely be disregarded."

During February, March, and April, 1916, experiments were made by Warrent R. Borst, Assistant Engineer, under Theodor S. Oxholm, M. Am. Soc. C. E., Engineer in Charge, Bureau of Engineering, Borough of Richmond, to determine the absorption of atmospheric oxygen by dilutions of sewage in fresh and salt water.* The liquid was placed in one of four tanks 25 ft. long, 6 ft. wide, and 6 ft. deep, and tested at stated intervals for dissolved oxygen (*a*) when quiescent; (*b*) when slightly agitated by two electric fans; and (*c*) (in the saline mixtures) when mechanically agitated by a strip of furring extending across the tank forming waves about 4 in. high. Temperatures were maintained at about 20° cent. by exhaust steam and simultaneous samples were taken at different depths.

As a control, tightly stoppered bottles of the mixture were kept immersed in the tank during agitation, by which the oxygen demand could be determined and allowed for. The absorption was then computed by adding the oxygen demand to the increase of dissolved oxygen in the tank.

These experiments were never carried out to such an extent that specific rates of absorption could be confidently stated, but they warranted the acceptance of certain general conclusions. With twenty dilutions of sewage with fresh water and a ½-in. ripple the absorption of oxygen after the fourth hour was 0.18 parts per million per hour, whereas with a like dilution with salt water it was 0.085 parts per million per hour. With a salt-water dilution devoid of oxygen at the start and a 4-in. wave the dissolved oxygen at the

* Report on Sewage Experimental Investigations at West New Brighton, Staten Island, N. Y., by Warren R. Borst, 1919.

end of 4 hours at all depths was 6.56 parts per million—an increase of 1.64 parts per million per hour. After 4 hours' agitation and 20 hours' rest, the dissolved oxygen had decreased to 4.50 parts per million. In general, it was found that "reaeration is decidedly rapid and uniform throughout the body of water."

Experiments along somewhat similar lines were made by Professor Edward Ellery, of Union College, and the results given in evidence before the Supreme Court in the Passaic Valley sewer case.* These comprised studies with polluted and unpolluted fresh and salt water, both quiescent and in motion. The results reached by Professor Ellery may be summarized as follows: When it is 40 to 60% saturated with dissolved oxygen, water of 60 to 70% as much salinity as sea water absorbs atmospheric oxygen at about the same rate as fresh water. This rate varies from between 0.18 cu. cm. and 0.306 cu. cm. per liter per day in quiescent water to 0.515 cu. cm. in water flowing at the rate of 1 ft. per sec. It diminishes as the humidity increases and increases with increased pollution. Under conditions of low humidity a permissible dilution of sewage in water quiescent or moving less than 1 ft. per sec. is 1 to 100, while a dilution of 1 to 60 is not permissible, resulting in an appreciable lowering of the oxygen content.

Standard of Cleanness.—Suggested standards of cleanness for New York Harbor have naturally laid stress on the dissolved oxygen content. Maximum values have varied from 70% saturation, proposed by Messrs. Black and Phelps in order to avoid detriment to major fish life, to 25% proposed by the late Rudolph Hering, M. Am. Soc. C. E., who stated: "I consider a standard of 25% with frequent sludge removal from the harbor bottom a safe protection against any nuisance arising from the water." Opinions of other authorities range between these limits. In a review of the subject† the speaker gave as his opinion:

"That to ensure a supply of dissolved oxygen at all points of a stream or harbor a liberal margin is required in the main body of the water, which, in the case of New York, should not be less than from 30% to 50% saturation, depending upon the intensity of local pollution due to sewage, the septicity of the sewage, the temperature of the air and on the amount of the sludge deposits on the bottom."

The essential matter is to provide that at no point shall the water be deprived of all its dissolved oxygen. Owing to the difficulty in expressing this in definite terms that might not be misconstrued, the Metropolitan Sewerage Commission finally omitted reference to any specific degree of saturation from its recommended standard which was as follows:

"PROPOSED STANDARD OF CLEANNESS

(1) "Garbage, offal, or solid matter recognizable as of sewage origin shall not be visible in any of the harbor waters.

* Transcript of Record, October Term, 1916, No. 3, Original. The People of the State of New York *vs.* the State of New Jersey, and the Passaic Valley Sewerage Commissioners, Vol. 3, p. 2824, and Vol. V, p. 4603.

† Annual Report, Chief Engineer, Board of Estimate and Apportionment, New York, N. Y., 1917.

(2) "Marked discoloration or turbidity, effervescence, oily sleek, odor or deposits, due to sewage or trade wastes, shall not occur except perhaps in the immediate vicinity of sewer outfalls, and then only to such an extent and in such places as may be permitted by the authority having jurisdiction over the sanitary condition of the harbor.

(3) "The discharge of sewage shall not materially contribute to the formation of deposits injurious to health or navigation.

(4) "The quality of the water at points suitable for bathing and oyster culture should conform substantially as to bacterial purity to the drinking water standard. It is not practicable to maintain so high a standard in any part of the harbor north of The Narrows, or in the Arthur Kill."

No further attempt has been made to formulate a standard.

Plans for Remedy.—The general principles underlying any reasonable plan of main drainage are set forth in Part II, Chapter I, of the Final Report of the Metropolitan Commission, which proceeded to describe in some detail a plan that it could recommend. Similarly, Messrs. Black and Phelps, in their report to the Chief Engineer of the Board of Estimate and Apportionment of New York, previously mentioned, presented a general scheme for the collection and disposal of the sewage, while, in 1920, the writer also presented a report to the Chief Engineer of the Board reviewing the entire subject and submitted a tentative revision of the general plan.* Based on these earlier studies and taking advantage of the more recent developments in methods of treatment, the tentative plan as now proposed to meet 1960 conditions may be briefly stated, as follows:

Where a high degree of stability as well as freedom from solids is demanded, the activated sludge process promises to afford the most desirable form of treatment, and plants of considerable size are contemplated: (1) At Wards Island, to handle 187 000 000 gal. per day of sewage; and (2) south of Jamaica to handle 114 000 000 gal. per day. These, it is believed, will control the serious pollution that would otherwise occur in the neighboring parts of the East River and Jamaica Bay.

Large populations on other areas, tributary to the East River, Jamaica Bay, and the waters of Coney Island, will make it necessary to remove a much larger quantity of polluting material than can be accomplished by screening, and sedimentation plants will probably be installed in the East River at Old Ferry Point, Tallman's Island, Rikers Island, Welfare Island, near Sheepshead Bay at the precipitation plant now known as Caisson No. 4, and on Rockaway Point.

For other localities it is believed that fine screening will suffice. Of these screening plants the most important now contemplated will be at West 155th Street, Manhattan, to which about 92 000 000 gal. per day will be diverted from areas now tributary to the Harlem River and carried to a submerged outlet 1 000 ft. or more from shore. If this plan should be carried out it would be equivalent to a reduction in population on the tributary drainage areas roughly estimated at 1 180 000 in 1960, or 33% of the resident population, as follows:

* Annual Report, Chief Engineer, Board of Estimate and Apportionment, New York, N. Y., 1920.

Assuming a removal of 5% of the putrescible matter by fine screening, 33⅓% by sedimentation, and 85% by the activated sludge process, the results would be:

5%	of 1 992 000 persons by screening	....	100 000
33⅓%	of 342 000 persons by sedimentation	....	181 000
85%	of 1 052 000 persons by activated sludge	......	899 000
	Equivalent to a reduction in population of	....	1 180 000

Stated another way, the result would be the same as if the population were decreased from 3 586 000 to 2 406 000. The estimated cost of carrying out this project, including interception, is $33 442 000, or $28 341 for the elimination of the pollution resulting from each 1 000 persons.

Most of these projects have never advanced to the point of authorization or even of general discussion and acceptance. It is probable that they will, in large measure, be adopted and the works built piecemeal as demanded, but modified as to detail to meet the situation. It is hardly open to argument (*a*) that to protect the waters of the Lower East River and Jamaica Bay, either a large volume of the sewage must be entirely removed or else some method of treatment equivalent to the activated sludge process must be used; and (*b*) that, for the further protection of the large volumes of sewage that will eventually reach the East River, other large volumes will have to undergo some more complete treatment than fine screening, but for the majority of the smaller installations this will be all that can be reasonably required. Chlorination is desirable where bathing beaches or shellfish are to be protected, but otherwise would appear to be an unwarranted expense.

Progress already made in carrying out this program includes the following items:

On the Hudson River, a fine-screening plant with two 14-ft. Riensch-Wurl screens, chlorination, and submerged outlet has been installed at Dyckman Street, Manhattan.

A fine-screening plant with three 6 by 55-ft. Rex screens and chlorination has been installed at Canal Street, Manhattan, for the Clarkson Street sewage brought to the plant by a 54-in. intercepting sewer.

On the Upper East River, a fine screening plant with two 14-ft. Riensch-Wurl screens, chlorination, and a 36-in. submerged outlet, 3 280 ft. long, has been installed at 25th Street, near North Beach, Queens.

On Staten Island, a fine-screening plant with two Dorrco screens, 6 ft. in diameter by 4-ft. face, with chlorination and a submerged outlet was installed in 1923. This is operated in the summer for the protection of bathers at South Beach and Midland Beach.

A fine-screening plant with two Link-Belt screens, chlorination, and a submerged outlet to protect a bathing beach near Great Kills is under construction.

On the shores of Jamaica Bay at the old 26th Ward Plant, Brooklyn, two 14-ft. Riensch-Wurl screens which have been in operation for several years and two new 26-ft. screens of the same type have been installed in the building that heretofore has housed the tanks of a precipitation plant.

In Queens, a plant comprising two 26-ft. Riensch-Wurl screens is being constructed south of Jamaica to serve the Fourth Ward of that Borough (north of Jamaica Bay) and one with two 22-ft. Riensch-Wurl screens at Hammel to serve the Fifth Ward (The Rockaways). The former constitutes the first installation of an activated sludge plant to treat 120 000 000 gal. per day of sewage from the entire Fourth Ward of Queens.

These fine-screening plants (not to mention several temporary hand-operated screens in Queens) probably handle about 85 000 000 gal. per day of sewage—a small quantity, considering the outlay—but at most of the plants the increase in volume will be rapid. In the greater part of the city the main lines of development are fairly well determined. The two major remaining problems are those relating to the sewage tributary to Wards Island and that of South Brooklyn. At present, Wards Island, although belonging to Manhattan, is leased to the State for institutional purposes until 1963, but it is hoped to obtain a release of about 50 acres through legislative action. This will clear the way for initiating the work of interception and treatment—a measure of great sanitary importance to the city.

The entire subject of handling the sewage of Brooklyn and Queens tributary to Jamaica Bay was treated in a report submitted by the writer to the Chief Engineer* in 1917. At that time a plan originating with the Metropolitan Sewerage Commission was recommended, conveying all the sewage tributary to the Bay from the north to a treatment plant on Barren Island. Since then developments at Barren Island and Rockaway Point to the south and a greater confidence in the reliability and inoffensive character of the activated sludge process have resulted in a modification of the original plant,† whereby the sewage of the Fourth Ward, Queens, will be treated locally by this process and the sewage of Brooklyn—at least from Coney Island to Canarsie—will be brought to an old existing precipitation plant near Sheepshead Bay, known as Caisson No. 4, for some suitable method of treatment. The sewage tributary to the 26th Ward Plant may later be treated by activated sludge or, more probably, brought to Caisson No. 4 for treatment.

To provide a central authority to co-ordinate interborough problems of sewage disposal and the control of harbor pollution a Committee on Main Drainage has been appointed, consisting of the President of each of the five Boroughs of New York and the Chief Engineer of the Board of Estimate and Apportionment, Arthur S. Tuttle, M. Am. Soc. C. E. In a broad sense, these problems are of general concern, affecting the welfare of the city as a whole, and by the agency of this Committee the interests of the different Boroughs not only may be presented, but their importance to the entire subject of the city's sanitation may be correlated and unified.

* "The Main Drainage and Sewage Disposal of the Area Tributary to Jamaica Bay."

† The plan as proposed in 1917 for the Fifth Ward, comprising Far Rockaway and the Rockaway Peninsula, has since been adopted and is now under construction with a temporary screening plant and outlet at B. 87th Street, Hammel.

RELATION OF DEPTH TO CURVATURE OF CHANNELS

Discussion*

By Messrs. E. J. Walker and Hubert Engels.

E. G. Walker,† M. Am. Soc. C. E. (by letter).‡—It would seem that the author has developed his formulas into needlessly long statements. Both Equations (4)§ and (5)§ may be written in general terms:

$$Y = D\left(1 - \frac{X^2}{W^2}\right) + D\frac{a}{R}\left(1 - \frac{X^2}{W^2}\right)X$$

$$= D\left(1 - \frac{X^2}{W^2}\right)\left(1 + \frac{a\,X}{R}\right)$$

Putting $x = \frac{X}{W}$ and $r = \frac{R}{W}$

$$\frac{X}{R} = \frac{X}{W} \times \frac{W}{R} = \frac{x}{r}$$

Therefore,

$$y = \frac{Y}{D} = (1 - x^2)\left(1 + \frac{a}{r}\,x\right) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

It follows from Equation (8) that as the curvature, $\frac{1}{R}$, to which $\frac{a}{r}$ is directly proportional, is increased, the value of y for a given value of x is increased; that is, increased curvature makes increased depth. According to Equation (8) there is no limit to this, a conclusion which is at variance with the first result given by the author.‖

Examination of the cross-sections plotted in the paper shows that in the great majority of the curved channels the actual erosion on the outer bank and accretion on the inner are more than is indicated by the profile plotted from the author's formulas. On the other hand there is a very close agreement indeed between the calculated and actual maximum depths and their positions relative to the center line of the cross-section.

These considerations lead to the suggestion that it might be possible to amend the formulas by the introduction of additional terms which would make a still closer approximation between the calculated curve and the actual profile, but which would not have appreciable effect on the calculated maximum depth

* This discussion (of the paper by H. C. Ripley, M. Am. Soc. C. E., published in December, 1925, *Proceedings*, and presented at the meeting of February 3, 1926), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† London, England.

‡ Received by the Secretary, January 21, 1926.

§ *Proceedings*, Am. Soc. C. E., December, 1925, Papers and Discussions, p. 1909.

‖ *Loc. cit.*, p. 1929.

or area of cross-section. No doubt the author has tried a number of approximations before arriving at his formulas, but, unfortunately, he gives little clue to the reasoning which has led him to adopt the particular expressions given. It is a little difficult, therefore, to estimate the possibilities of getting a closer approximation.

DR. HUBERT ENGELS* (by letter).†—The practical value of Mr. Ripley's formulas lies principally in the possibility of determining the "cross-profiles" of "outer bar channels". The investigations of the mouth of the Rio Grande do Sul and the Southwest Pass of the Mississippi with the aid of these formulas have shown the superiority of a single curved jetty, not only over two straight parallel jetties, but also over two parallel curved jetties. This is new and important knowledge.

The writer is fully in accord with the author that his formulas are "of the greatest value in the regularization of rivers and in the improvement of outer bar channels". He cannot agree with him, however, that any attempt to deepen "straight reaches" in estuaries by dredging would destroy the "tendency to uniformity of depth" persisting there; that it, therefore, would not seem practicable to secure "permanent improvement in the navigable channel" by dredging, nor that it would be much more logical "to convert the straight reaches into channels of suitable curvature by means of training walls and to let Nature do the rest". Certainly, dredging alone is no method of regulation, as it attacks only the result without removing the cause. Nevertheless, one should not leave the intended deepening of the channel by works to the influence of the works alone, but should bring it about as far as possible by dredging; if the works (as, for instance, "training walls") are properly placed, their influence consists in the maintenance of the dredged channel.

If, in accordance with the author's proposal, a straight channel in estuaries were transformed into a curved one, the force of the tide wave would be lessened, a part of its energy wasted, and the quantity of water flowing in and out decreased, so that the scouring force of the stream would be weakened. Therefore, the retention of straight channels in estuaries is unobjectionable if these channels are so bordered by "training walls" that the scouring force of the river remains sufficiently great. Above the tidal region it is generally desirable to give the river such bends that the current is in a state of inertia. Aside from this objection, however, the paper is to be welcomed with gratitude as a valuable advance in the still dark field of river hydraulics. In addition to M. Fargue and the late Mr. Mitchell, referred to by the author, M. Boussinesq‡ has also studied "the relation of depth to curvature".

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* Prof. Emeritus of Hydraulics, Technische Hochschule, Dresden, Germany.

† Received by the Secretary, January 29, 1926.

‡ "Essai sur la théorie des eaux courantes", Paris, Mémoires présentés par divers savants a l'Académie des Sciences, de l'Institut de France, Tome 23, 1877; Supplement 24 (1877).

NOTES ON SHEAR IN COMPRESSION MEMBERS

Discussion*

By Messrs. C. A. P. Turner and E. G. Walker.

C. A. P. Turner,† M. Am. Soc. C. E. (by letter).‡—The difficulty in adapting beam theory to the column is that such a theory, like a playful pup, chases its tail in a circle without practical progress. Thus, the fiber stress caused by bending varies as the bending moment; the deflection varies as the fiber stress due to bending; but for the same load, the bending moment varies as the deflection. Hence, the magnitudes of the fiber stress, deflection, and bending moment are theoretically unstable under the applied beam theory and the assumption that the bending is parabolic does not help in stabilizing sin G on which the author's calculations depend.

The amount that a column under load bends or deflects has no direct proportion to the amount of the load; nor, in fact, to the apparent eccentricity of the stressed area at mid-length; nor to the ratio of the mean compressive stress, u, to the maximum, f. However, when $\frac{f}{u}$ equals unity the eccentricity, e, is zero and the deflection also is zero.

When $\frac{f}{u}$ at mid-length equals 2 the area of fiber stress distribution is triangular and entirely compressive, that is, zero on the convex side and increasing uniformly to a maximum on the concave side. Relative deflections for a given ratio of $\frac{L}{R}$ may be plotted to illustrate typical column action, rating as unity§ the deflection when $\frac{f}{u}$ equals 2.

Such a curve of relative deflections shows that the solid column of uniform material obeys Hooke's law in its elastic lateral deflections, since these deflections are proportional to the product of two factors, namely, (1) the eccentricity, e, at mid-length of the total stressed area (the lever arm); and (2) the magnitude of the triangular (the force).

The lateral deflection of the column is a continuous function. The fact that the deflection is almost negligible at the critical load (under which the stress area changes from total compression to part tension) and reduces to but a minute fraction of this almost negligible amount under working loads, has given rise to the erroneous supposition that the deflection is a discontin-

* This discussion (of the paper by Ralph E. Goodwin, Assoc. M. Am. Soc. C. E., published in December, 1925, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., Minneapolis, Minn.

‡ Received by the Secretary, December 11, 1925.

§ "Elasticity and Strength of Materials", Turner, Section IV, p. 8.

uous function absolutely instead of being approximately zero under ordinary working loads.

In test specimens, except those in which the slenderness ratio, $\frac{L}{R}$, is large, no tension parallel to the axis will exist in any of the fibers until the column may be said to have failed. The common conception of the bending of a column is erroneous, being founded on the impression obtained from the curve of flexure used in deriving the formula, which indicates a considerably bent member, with a tension and compression zone representing the conditions long after the maximum load has passed and the deflection has greatly increased under reduced load in the follow-up action of the testing machine, thus giving a false impression of the true action.

Because the safe unit stress decreases as the slenderness ratio increases various unsymmetrical rolled sections, such as angles, T's, or double angles, are used in roof trusses and H-sections, Z-bar columns, plate and channel sections, etc., for larger sections in building and bridge work. If one of these hollow unsymmetrical sections be divided in longitudinal strips, each presents a slenderness ratio of a far higher order than that of the section as a whole and hence the total resistance becomes some function of the particular combination of strips presented by the configuration of the section, requiring investigation of different typical forms to determine how the formulas for the solid section should be modified for application in design.

A strip at the extremity of an angle, T, or Z-section, is relatively lacking in rigidity laterally and will tend to buckle out of line normal to the direction in which the strut tends to bend under axial compression. Such a buckling tendency gives rise to the twisting action presented by the Z-column and other sections when tested to destruction. The extent of this torsional stress should be investigated in addition to the bending action typical of the column of solid symmetrical section.

Twisting of the column about its axis under axial compression cannot occur in a solid shaft, such as a round or a square column, because the resulting deformation increases the length of the shaft, which, under a force tending to shorten it, is contrary to Hooke's law. The failure of the solid column, therefore, is one of pure crushing and bending; and the same reasoning applies to the hollow cylinder. In the unsymmetrical section or hollow section other than a cylinder the phenomenon of twist occurs, and for a given twisting (in this case the lateral buckling tendency of the strips) the angle of distortion increases as the free length of the shaft twisted. For a column with round ends the full length of the shaft may contribute to this deformation, but if the ends be pin-connected or clamped they are fixed against torsional rotation at the ends and the free length for the summation of the twist is diminished to one-fourth the length of the column.

Mr. Christie's tests* show the strength of an angle-iron strut with ball-and-socket ends having a slenderness ratio, $\frac{L}{R}$, greater than 140 to be four-

* "Elasticity and Strength of Materials", Turner, Table, Section IV, p. 27, and "Mechanics of Material," Merriman, 1916 Edition, p. 198.

sevenths as great as one with pin ends; but the pipe column has identically the same strength whether it is* pin-ended or† has ball-and-socket bearings. For the same ratio, $\frac{L}{R}$, a pipe of the same area and the same material is substantially one-seventh stronger than a corresponding angle, pin-ended, and with ball-and-socket bearings the pipe is twice as strong as an angle when the equal slenderness ratios are greater than 140. Calculated according to column theories which have been derived from the erroneous application of beam theory the three kinds of struts have the same theoretical strength although they differ 75 to 100% in actual strength.

No wonder that German engineers, after adopting sections for their bridge structures which would readily twist, strove to find a new theory of action of the main members that would account for the failures. They accordingly developed a theory of secondary stress instead of rectifying their theory of primary stress. They have inflicted on the profession the illogical secondary stress computations which have no practical value when the primary stresses are rationally provided for and when suitable triangular elements are adopted for the frame.

Only the minimum radius of gyration of a member is considered in the ordinary column formula, yet the maximum radius of gyration has a marked effect as regards the manner in which the section fails under compression. The Z-bar column having the same radius of gyration in both directions, if it is of minimum section, twists like a corkscrew before failure. On the other hand, the same Z-section, if the Z's are spread so that one radius of gyration is much greater than the other, will fail as they did in Strobel's tests,† by pure bending and crushing.

In large columns the symmetrical section will develop far higher resistance than those commonly used in American bridge design. The column sections in the Forth Bridge present a satisfactory distribution of metal, but the work involved was excessive. To obviate this difficulty the writer has devised a stave column, octagonal in form, with covers inside and outside of the apices, using cast-steel rings at intervals to maintain the symmetrical shape of the section. Such a section is more readily spliced than those in ordinary use. It lends itself readily to the connection of riveted web members and lateral struts, and can be made accessible for painting and inspection inside as well as out.

The collapse at Quebec was the last step in the practice of saving weight of pin-plates by concentrating the metal in the cross-section of the web. The chord sections differed little from those of the Blackwell Island Bridge and of other bridges except in the seemingly conservative increase in the slenderness ratios, but this increase gave greater opportunity for twisting and buckling of the built channels, which were sadly lacking in stiffness in comparison with the rolled sections usual in the laced channel compression bridge member. The laminated webs were not comparable to the solid web of the rolled channel while the flanges, the function of which is stiffening the web against buckling,

* See, plot, Watertown Tests, Salmon columns, p. 219.
† *Transactions*, Am. Soc. C. E., Vol. XVIII (1888), p. 103.

were of negligible area and small width. The need of correspondingly substantial bracing and connections of the bracing was assumed to be unnecessary. Thus, a forceful object lesson in the need of applying the principles of proportion from models of known strength and resistance is brought home by this failure.

E. G. Walker,* M. Am. Soc. C. E. (by letter).†—The author has done service not only in applying mechanical principles to the determination of shear stresses in compression members, but also in emphasizing the necessity for calculating such stresses in many cases that occur in practice. This necessity is overlooked frequently; in many sound textbooks on the mechanics of structures no treatment of the problem is attempted or even mentioned.

The basis of the treatment developed in the paper is not new. The writer has found it in textbooks with the modification that the deflection curve of the compression member has been taken as a sinusoid instead of a parabola. There does not appear to be much difference in the two treatments except that the author's assumption produces some simplification in the equations and arithmetic.

The author is right in deprecating the method of determining shear by calculations based on the assumption of an equivalent uniformly distributed lateral load. There seems to be no connection between the internal forces set up in a compression member and in the presumed equivalent beam except that the two have the same maximum direct fiber stress. Surely it is making a very broad assumption indeed, to consider that because of these equalities the maximum shear is the same in both cases. It is time that such a basis of calculation should be abandoned especially as the rational method is so easy of application.

There is some analogy between the present problem and the kindred one of determining the web stresses of a plate girder. Various standard specifications give different empirical ways of calculating the shear which is to be taken as an average and which must fall below a prescribed maximum. The quantity thus calculated is not a maximum stress and there is no reason why the actual maximum shear stress in a girder should not reach an unsafe value although the calculation should fulfill the specification; it is only a question of distribution of material. It is easy to calculate the actual shear stress at various critical points of the cross-section by rational methods and by the application of principles that are explained at an early stage in the teaching of structural mechanics. There is little justification, therefore, for the use of empirical or semi-empirical methods in this case, and the same argument applies to the problem considered in the paper.

The author's illustrative calculation is possibly the most useful feature of the paper, as it draws attention in a very definite way to the risks inherent in the use of the equivalent beam method of calculation and cuts away the support often given to that method by illustrative examples, in which the shear

* London, England.

† Received by the Secretary, January 14, 1926.

calculated by it is shown to be considerably greater than that from direct calculation. The author shows that the reverse may easily be the case to a dangerous extent. The writer hopes, therefore, that this paper will draw attention to the defects of the equivalent beam method of estimating shear stress in compression members and cause more exact methods to be substituted in standard specifications.

MEMOIRS OF DECEASED MEMBERS

Note.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

EDMUND HAMILTON BOWSER, M. Am. Soc. C. E.*

Died May 8, 1925.

Edmund Hamilton Bowser was born at Louisville, Ky., on January 13, 1856. His father was John W. Bowser, formerly of Baltimore, Md., and his mother before her marriage was Anna Culver, of New Orleans, La. She was gifted as a poetess and wrote a prize-winning poem for the Nashville Centennial.

Mr. Bowser received his engineering education at Vanderbilt University, Nashville, Tenn., which he attended from 1878 to 1882. At the time of his graduation he was awarded a scholarship for the highest standing in the engineering class. For a year after graduation he remained at the University as Physical Instructor.

In 1883 he entered the service of the Cincinnati Southern Railway Company as Assistant Engineer, and continued in this position for two years. For three years thereafter he was in Florida, engaged in general engineering work. On April 15, 1888, he entered the service of the Newport News and Mississippi Valley Railroad Company as Assistant Engineer. This railroad later became the Chesapeake, Ohio, and Southwestern, and is now a part of the Illinois Central System. While with the Chesapeake, Ohio, and Southwestern Railroad Company he was stationed at Louisville, Ky., with the title of First Assistant Engineer, and was in charge of the work in the Bridge, Building, and Track Departments.

In 1898, a year after the Chesapeake, Ohio, and Southwestern Railroad was incorporated with the Illinois Central System, Mr. Bowser was made Roadmaster, with headquarters at Memphis, Tenn. He resigned this position in July, 1899, to become Assistant Engineer with the Louisville and Nashville Railroad Company, with headquarters at Pensacola, Fla. While in this position he had charge of several large dock projects. In October, 1903, he left the employ of the Louisville and Nashville Railroad Company to become General Manager of the Southern Creosoting Company, at Slidell, La.

On August 1, 1907, Mr. Bowser re-entered the employ of the Illinois Central Railroad Company as Chief Timber Inspector and on January 1, 1912, was promoted to the position of Superintendent of the Timber Department. On January 1, 1922, he became Superintendent of Ties and Treatment, which position he held until his death.

He was an active member of the American Railway Engineering Association and the American Wood Preservers' Association. He had served as a member of the Committee on Wood Preservation of the American Railway Engineering Association for about fifteen years, having been Vice-Chairman in 1912, 1913, and 1914. In the last report of this Committee,† Mr. Bowser contributed a paper entitled "The Effect of Preservatives on the Inflammability

* Memoir prepared by A. L. Dabney, M. Am. Soc. C. E.

† *Bulletin 270*, Am. Ry. Eng. Assoc., October, 1924.

of Wood." He was a recognized authority on wood preservation and his papers on the subject attracted wide attention. For more than twenty years he made a close study of wood preservation, and it is largely due to his efficient and conscientious work that the present high standard of wood treatment has been attained.

Mr. Bowser was a Charter Member of the Engineers Club of Memphis, a member of its first Board of Directors, and during 1922 served as President. Under his guidance the Club was inspired with a new enthusiasm and grew in membership and influence. At the close of his administration he delivered an address which is notable in the Club's history. It was immediately ordered published for distribution, and has the distinction of being the only paper that the Club has put into print. The following quotation from this address has, since its utterance, appeared on every *Bulletin* of the Club:

"The true Engineer is one who sees visions, and following his visions beyond the beaten path of common practice, does something that has not been done before, or does it in a better way."

From his boyhood Mr. Bowser had been an active member of the Unitarian Church. For five years he was President of the Board of Trustees of the of the Memphis Church, and was Chairman of the Building Committee for the new church building erected in 1923.

He was married to Alice Fairfax Smith, daughter of Judge James Power Smith and Julia Robert Smith, of Chattanooga, Tenn., on June 20, 1889. He is survived by his wife, three nephews, Allen F. Owen, of Chicago, Ill.; and Wilfred and David Bowser, of Anchorage, Ky., and two nieces, Miss Ethel Owen, of Washington, D. C., and Mrs. Helen Owen Lee, of Pasadena, Calif.

Mr. Bowser was drowned with the sinking of the Steamer *Norman* in the Mississippi River, near Memphis, Tenn., on May 8, 1925. He had with him a great-nephew, a boy ten years old. A small power boat came to the aid of the *Norman's* passengers, and Mr. Bowser swam to the side of this boat and placed the boy in it, but refused to get in himself until the women and weaker swimmers were saved. He swam away to assist others and was not again seen alive.

His sincerity, optimism, enthusiasm, and geniality won and held his fellow engineers who had for him a strong esteem and genuine affection.

Mr. Bowser was elected a Member of the American Society of Civil Engineers on June 1, 1904.

GEORGE THOMAS FORSYTH, M. Am. Soc. C. E.*

DIED AUGUST 31, 1925.

George Thomas Forsyth, the son of Elizabeth and Jonathan Forsyth, was born in Providence, R. I., on January 19, 1876. In his early childhood his parents moved to Salinas, Calif., where his boyhood was spent and where he received his public school education.

In 1895 he entered Leland Stanford, Jr., University. There he pursued a special engineering course until the spring of 1899, when he accepted a

* Memoir prepared by S. Murray, M. Am. Soc. C. E.

position with the Union Iron Works at San Francisco, Calif., where he was engaged until December, 1900, in ship-fitting and the erection of marine engines.

In January, 1901, Mr. Forsyth entered the service of the Southern Pacific Company at San Francisco, as Draftsman in the office of the Engineer of Bridges. This was on the eve of the great physical development which was to take place in that railroad and in other lines then being associated with it by the late E. H. Harriman. In 1903, Mr. Forsyth was promoted to the position of Chief Draftsman, in which capacity he was in direct charge of bridge and miscellaneous structural and mechanical design. He remained in this position until 1905, during which period he took an important part in the preparation of the common standard plans of the Harriman System. These plans touched almost every physical detail of railroad construction and maintenance, and their value is evidenced by the fact that most of them are still in use on the Union Pacific and Southern Pacific Systems. Perhaps his most effective efforts were directed to the standard bridge designs, in which he showed ingenuity and clearness of thought that seemed sometimes to approach genius.

In 1905, Mr. Forsyth was appointed Bridge Engineer of the Oregon Railroad and Navigation Company and of the Oregon Lines of the Southern Pacific System. In the great program of extensions and improvements which had just begun, and during the next eight years, he was continuously occupied with the design and construction of many bridges, but he also found time to devote himself to power-plant and wood preservation problems. His most important work in this period was in connection with a large double-deck lift-bridge over the Willamette River, at Portland, Ore., which was constructed under his supervision, and to the design of which he contributed largely.

In 1913, Mr. Forsyth decided that better opportunities existed outside of railroad service, and with the regret and good wishes of his associates he joined the Union Bridge Company of Kansas City, Mo., and later the contracting firm of Diecks and Diecks. In these engagements he successfully carried to completion a number pneumatic caisson and subaqueous tunnel projects.

In 1920, Mr. Forsyth returned to Portland, Ore., where he opened an office and engaged in independent practice. Early in 1924, he was called to take charge of a tunnel beneath the Duwamish River at Seattle, Wash., in the construction of which difficulties had developed that appeared to be insurmountable. Under his direction, this project, which had seemed palpably a failure, was carried to a swift and successful completion. This was his last important work.

Mr. Forsyth was a brilliant mathematician and a leader in bridge design and construction. Loyal and steadfast, he was a man of independent thought and of original and whimsical humor. His passing leaves a gap which his friends and associates can never fill.

He was married on June 16, 1902, to Florence Ligon, who, with one son, George Thomas, Jr., survives him.

Mr. Forsyth was elected a Member of the American Society of Civil Engineers on October 1, 1912.

CHARLES STEWART MAURICE, M. Am. Soc. C. E.*

Died February 20, 1924.

Charles Stewart Maurice was born at Perth Amboy, N. J., on June 29, 1840, the son of Charles Frazier and Cornelia Joline Maurice. On his father's side, the family was of Welsh and Scotch-Irish descent, and on his mother's, a combination of Huguenot French and Holland Dutch, the latter having been among the earliest settlers of New Amsterdam.

In 1842, Charles Frazier Maurice, at the earnest solicitation of a number of friends, opened a small private school at Napanock, N. Y. Several years later, the growing school needing enlargement, was moved to Sing Sing (now Ossining), N. Y., and was merged with the old Mount Pleasant Military Academy, which became one of the best known private schools of its time. It was here that the son received his early education. In 1858, he entered Williams College at Williamstown, Mass., as a Sophomore, and was graduated in 1861, the Salutatorian of his Class, and a member of the honorary society, Phi Beta Kappa.

In order to serve his country in the Civil War which by this time had come to be considered as more than a "90-day affair," Mr. Maurice decided to enter the Navy. In order to render effective service, however, he desired to prepare himself for a Commission on the Engineer Staff, and began the study of Marine Engineering, entering Rensselaer Polytechnic Institute, Troy, N. Y., during the fall of 1861, where he covered the subjects allotted to the Sophomore and Junior years in one year. On November 17, 1862, he was commissioned Third Assistant Engineer (Midshipman), U. S. Navy, and reported for duty at the Washington Yard, where he was assigned to the *Ossipee*, which later joined the North Atlantic Squadron, doing duty on the blockade of the Southern States on the Atlantic seaboard.

Never having had robust health, the restricted ration available in those days brought on severe and prolonged attacks of indigestion, and in the summer of 1863, Mr. Maurice was ordered home on sick leave. He never completely recovered from the effects of this illness, and was subject to indigestion, more or less severe, for the remainder of his life. On rejoining the Navy in the autumn of that year, he was ordered to the Portsmouth Navy Yard, and assigned to the fitting out of the *Agawan*, one of the "90-day" gunboats, built "double ended" for service on the narrow, crooked rivers of the South. Before her entire completion the *Agawam* was ordered to sea in chase of a Confederate privateer known to be off the New England coast, and was very nearly lost in a gale off Cape Sable, Nova Scotia. After her refitting, she joined the North Atlantic Squadron in the spring of 1864, Mr. Maurice going with her as Second Assistant Engineer, having been promoted to that rank on March 23, 1864.

The spring and early summer of 1864 was spent by Mr. Maurice on the James River, where a small fleet of ironclads and gunboats was keeping open

* Memoir prepared by George H. Maurice, M. Am. Soc. C. E.

a line of communication with General Grant, and was under frequent fire from enemy batteries and sharpshooters along the banks, notably at Dutch Gap and Malvern Mill. He was then transferred to the steam frigate, *Colorado*, of the West Gulf Squadron, doing blockade duty while Admiral Farragut was fighting in Mobile Bay, the *Colorado* not taking part in the fight, as she drew too much water to cross the bar. It is interesting to note that the late Admiral George Dewey was a Lieutenant on the *Colorado* at this time. Mr. Maurice was later transferred to the flagship, *Malvern*, of the North Atlantic Squadron, but had returned to the *Colorado* when she took part in the second attack on, and the capture of, Fort Fisher, on January 15, 1865. He remained in the Navy until December 21, 1865, on which date he resigned and was honorably discharged after declining the appointment as Assistant Professor of Mathematics at the United States Naval Academy at Annapolis, Md.

The problems of rapid transit in New York, N. Y., were even then demanding solution; the railway termini—both the New York Central and Hudson River Railroad on West 30th Street, and the Harlem and the New Haven Railroads on Park Avenue, between 26th and 27th Streets, were far removed from the business center on Lower Broadway. Several lines of river steamers designed to run a "commuter's service" were planned to operate on both the East River and the Hudson River, and Mr. Maurice was employed as Engineer by the Lower Hudson Steamboat Company. He designed the engines for the two boats of that Company, the *Sleepy Hollow* and the *Sunnyside*, built during the spring of 1866. He then trained the engine-room crews of these boats, which proved to be the fastest boats on the river with the exception of the famous *Mary Powell*, a much larger boat. They ran from the Battery, New York, to Sing Sing (now Ossining) and Peekskill, N. Y., but needless to say rapid transit around New York by water was not a success, and the Company went into bankruptcy. It was probably during the latter part of the winter of 1865-66 that Mr. Maurice found time to work for about two months in a foundry and machine shop at Peekskill, thus acquiring shop experience.

During the fall of 1866, he became interested, with his boyhood friend, Eugene Underhill, in the tanning business, and they spent the winter of 1866-67 at Liberty, N. Y., and also at Great Bend, Pa., investigating and observing the operations of the large tanneries in those sections. In the spring of 1867, they built their tannery from the plans of, and under the supervision of, Mr. Maurice, at Athens, Pa., this section of Bradford County then being in the center of large forests of original growth of hemlock, the bark of which was necessary for their operations. In the autumn of 1869, he disposed of his interest to his partner, and returned to the family home at Briarcliff, N. Y.

During the next few years the little village of Athens, situated on the banks of the North Branch of the Susquehanna and the Chemung Rivers, instead of being at the junction of the Pennsylvania and New York State systems of canals (now abandoned in that section), was in the close vicinity of the junction of four railroads. This rapid transition from water to rail transportation had been brought about by the work of the late Col. C. F. Welles,

a man of vision, sprung from the best pioneer stock in the valley of the North Branch. This change appealed so strongly to the business sagacity of the late Mr. N. C. Harris, a leading merchant and banker of this section, that he introduced Mr. Maurice to the late Charles Kellogg, M. Am. Soc. C. E., who had a patent on combination wooden and iron bridges and having already started works at Athens for their fabrication, was desirous of extending them. Mr. Maurice secured a contract from the old Oswego Midland Railroad covering timber bridges in which more than 1 000 000 ft. of timber was used, and afterward, in 1871, the partnership of Kellogg and Maurice was formed. An office was maintained in New York, of which Mr. Maurice was in charge until 1874, when he moved to Athens with his family, and there he continued to reside until his death.

The firm of Kellogg and Maurice was one of the pioneers in the building of iron bridges, and the second to build those of steel. Its first bridge of any note was one over the Tombigbee River in Southern Alabama, with a draw-span which was erected without falseworks, an unprecedented feat at that time, and one which was the subject of a paper* presented to the Society by Mr. Maurice on December 17, 1873. Among other early works of this firm may be mentioned a section of the Third Avenue Elevated Railroad in New York, noteworthy for the methods of erection, bridges in Nova Scotia and Brazil, one over the Snake River, in Idaho, and one over the Platte River at Plattsmouth, Nebr. Later, came the Thames River Bridge, at New London, Conn., with the longest swing-span of its day, 500 ft., double-tracked, but a pigmy beside present-day structures.

The partnership continued until 1884, when a merger was formed with other firms in the same business, and the Union Bridge Company was organized. The firm was composed of Charles Macdonald, Past-President, Am. Soc. C. E., formerly of the Delaware Bridge Company; the late Thomas C. Clark, Past-President, Am. Soc. C. E., formerly of Clark, Reeves and Company; the late Edmund Hayes, M. Am. Soc. C. E., and George S. Field, M. Am. Soc. C. E., of the Central Bridge Company; the late Charles Kellogg, M. Am. Soc. C. E., and Mr. Maurice. Shops were maintained at Buffalo, N. Y., as well as at Athens, Pa., and the Company became internationally famous for the number and importance of the bridges it built in all parts of the world, and for the excellence and integrity of its work.

Pin-connected bridges were the standard in those days. With the introduction of steel, special methods and plant became necessary and all these requirements were met by the partners with either original designs or improvements on tools already in use. As an instance, for the famous Hawkesbury River Bridge, in New South Wales, the Bessemer steel for the eye-bars came from England and was manufactured into bars at Athens; before they were accepted, however, these bars had to be annealed in special furnaces built for the purpose; and full-sized tests were made in the 600-ton hydraulic testing machine also built at Athens for this purpose. The sinking of the

* "An Account of the Erection of a Draw-Bridge without False Works", *Transactions*, Am. Soc. C. E., Vol. XI (1873), p. 330.

caissons for the piers of this bridge in the rapid tidal flow of the river was probably the most hazardous construction work undertaken by the Company; the spans were floated into position on pontoons.

Other noteworthy bridges built by the Company were the Poughkeepsie Bridge over the Hudson River, the Cantilever Bridge over the Niagara River, the Cairo Bridge over the Ohio River, the Memphis Bridge over the Mississippi River and many other bridges over the Mississippi, Ohio, and Missouri Rivers.

In 1895, Mr. Maurice was obliged to retire from business on account of ill-health, and spent the summer of 1896 in Europe. He continued to reside in Athens, taking daily drives over the hills of which he was so fond, and spending the winters at Jekyl Island, Georgia.

On April 28, 1869, at St. George's Church, New York, he was united in marriage to Charlotte Marshall, daughter of John G., and Marian Marshall Holbrooke, both of whom were descended from the oldest Pilgrim and Puritan stock of New England. Of this union nine children were born: Archibald S., Marian B., and Margaret S., residing at Athens, Pa.; George H., of Eagle Springs, N. C.; Charles F., of Glen Ridge, N. J.; Cornelia (Mrs. Robert Wilkinson), of Poughkeepsie, N. Y.; Albert T., of Rye, N. Y.; Emily M. (Mrs. C. W. Dall), of Cedarhurst, N. Y.; and Charlotte, who died in infancy. Mrs. Maurice, who had endeared herself to the community by her many kind deeds and acts of charity, and was beloved by all who knew her, died in 1909. Besides the sons and daughters, there survives a brother, Benjamin Maurice, of Mamaroneck, N. Y., and eleven grandchildren.

Of a retiring and quiet disposition, never voluntarily speaking of himself, Mr. Maurice's outstanding characteristics were absolute accuracy of mind and justice to all by thought and deed. He was a lover of Nature, and sought his pleasure and recreations in the woods and fields, as botanist, trout fisherman, wing shot, or as rider or driver, as the seasons and chance offered. During his early manhood, the flights of wild ducks and geese were especially alluring, and many a shooting trip was planned in connection with the building of bridges over Western rivers. In middle life, fly-fishing claimed a larger share in his recreation, and his usual summer vacations were spent somewhere in the wilds of Canada, accompanied by one or two of his sons, where his delight and pride was to land a 4-lb. trout with a 4-oz. rod from swift water. He rode horseback until he was over seventy years of age. To the time of his final illness he kept up his daily drives with fine horses, driven by himself until failing strength compelled him to relinquish the reins, and afterward by automobile.

At the time of his death, he was a member of the Loyal Legion, the Phi Beta Kappa Society, President of the Spaulding Memorial Library, of Athens, member of the American Forestry Association, the Jekyl Island Club, and the University and Union Clubs of New York City.

Mr. Maurice was elected a Member of the American Society of Civil Engineers on May 15, 1872.

EDMUND BOYD ULRICH, Assoc. M. Am. Soc. C. E.*

DIED NOVEMBER 20, 1925.

Edmund Boyd Ulrich was born at Reading, Pa., on September 25, 1871, the son of Dr. Daniel and Mary (Boyd) Ulrich. He received his education in the public schools of his native city, and was graduated from High School in 1885.

Immediately after his graduation, Mr. Ulrich entered the service of William H. Dechant, Civil Engineer, and after serving successively as Rodman and Transitman for several years, accepted a position as Transitman on a railroad construction operation in North Carolina. He then returned to Reading and served as Instrumentman on the construction of the Neversink Mountain Railroad, the Mohnton and Adamstown Electric Railroad, and other local construction operations.

In 1894, Mr. Ulrich entered the employ of the City of Reading, as Transitman, attached to the City Engineer's Staff, and served in this capacity during the construction of the trunk line intercepting storm sewer, and further work for the initial district of municipal sanitary sewers, including a pumping station.

In 1903, he established an engineering office, and engaged in surveying and general engineering practice until April 12, 1909, the date of his appointment to the office of City Engineer of Reading, for a term of three years. He was re-appointed to this office, successively in 1912, 1916, and 1920, serving continuously for fifteen years until May 7, 1924. Mr. Ulrich then re-established a surveying and engineering office, and in conjunction therewith served as City Engineer for Mahanoy City, Pa.

He was actively affiliated with and served as Secretary of the American Turbit (pigeon) Club. He owned select pigeons and enjoyed a National reputation as a pigeon fancier and competent judge, thus becoming a factor in promoting pigeon and poultry exhibits.

Mr. Ulrich was in ill-health for seven weeks before his death on November 20, 1925. He is survived by his wife who was Bessie Chamberlain, of Winston-Salem, N. C.

Mr. Ulrich was elected an Associate Member of the American Society of Civil Engineers on April 6, 1909.

* Memoir prepared by E. Clinton Weber, Esq., Reading, Pa.